POWER TRANSFORMER

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TRANSFORMER CONSTRUCTION

A transformer consists of two electrically isolated coils mutually coupled by a common magnetic circuit.

The magnetic circuit is laminated to reduce eddy current power losses.

Refer to FIG 1 which shows the construction of a basic transformer.

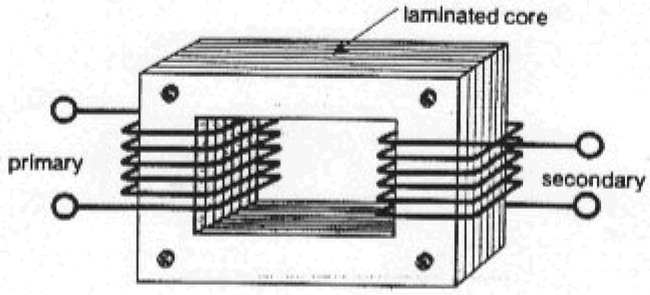

FIG 1

The arrangement of coils shown in FIG 1 is not ideal because there will be a leakage of magnetic fl
because the windings are not closely coupled.

The magnetic coupling between primary and secondary windings is improved by winding the coils
top of each other, with suitable insulation between the windings.

There are two types of transformer construction namely "Shell Type" and "Core Type" construction.

Shell Type Construction

Refer to FIG 2 which shows the arrangement of windings on the magnetic core for a "Shell Type" transformer.

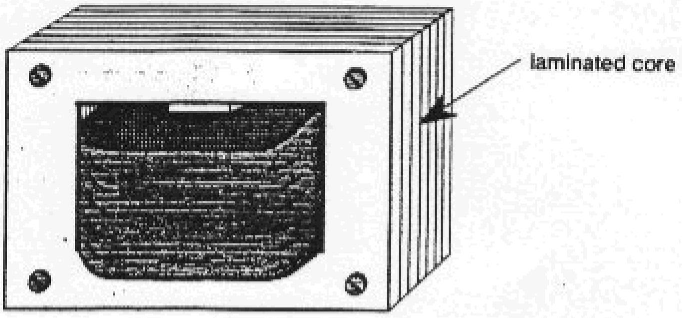

FIG 2

Refer to FIG 3 which shows the winding details of a "~~Core~~ Shell Type" transformer.

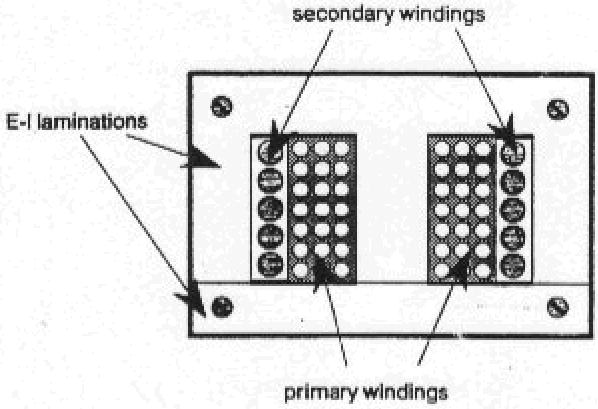

FIG 3

Core Type Construction

Refer to FIG 4 which shows the arrangement of windings on the magnetic core for a "Core Type" transformer.

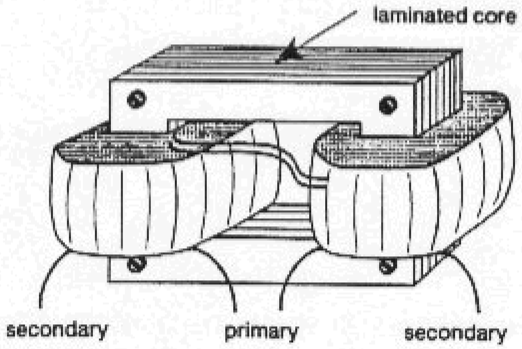

FIG 4

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Refer to FIG 5 which shows the winding details of a "~~Shell~~ CORE Type" transformer.

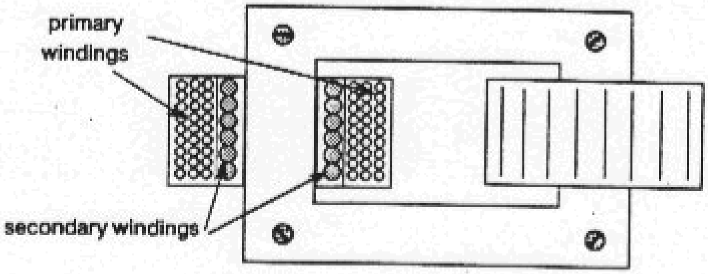

FIG 5

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TRANSFORMER RATINGS

Voltage Rating

Rated voltage or "Nominal" voltage of a transformer winding is the RMS value of the highest continuous voltage that can be applied to that winding.

Example: 2000/200V rating gives Vp and Vs RMS values

Notes: Iron losses are proportional to V^2, and flux is proportional to the voltage applied.

It is important that the voltage applied should not exceed the rated value otherwise magnetic saturation of the core may occur and there will be excessive iron losses.

Excessive voltage can result in breakdown of insulation on windings between turns and between windings and earth.

Current Rating

Rated current or "Nominal" current of a transformer winding is the RMS value of the highest continuous current that can be carried by each winding without causing excessive temperature rise.

Note: Winding power losses (copper losses) are proportional to I^2, and result in winding temperature rise, causing damage to windings and deterioration of insulation.

Power (VA) Rating

Power (VA) rating of a transformer is equal to the nominal voltage multiplied by the nominal current.

Power Rating = Vnominal x Inominal Volt Amperes

Notes: This rating is in units of VA and **not** Watts since the power factor (cos θ) of the load on the transformer can vary.

The power handling capacity of a transformer is expressed in VA, kVA or MVA depending on the size of the transformer.

Calculation of Rated Current

VArated = Vrated x Irated

Irated = VArated/Vrated

Example: Calculate the rated primary and secondary currents of a 400/200V 4kVA transformer.

Iprimary rated = VArated/Vprimary rated = 4000/400 = 10A

Isecondary rated = VArated/Vsecondary rated = 4000/200 = 20A

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Tut 1

Q1

a. number of primary turns

$$\frac{6600}{400} = 16.5 \text{ ratio}$$

$$V_{pt} = 15v$$

$400 \div 15 = 27$

$27 \times 15 = 405$

Q2. $N_P = 80 \Rightarrow 240v$

$N_S = 400$

$CSA = 200cm^2$

a) induced emf in secondary winding

$$\frac{E_P}{E_S} = \frac{N_P}{N_S}$$

$$\therefore E_S = \frac{E_P N_S}{N_P}$$

$$= \frac{240 \times 400}{80} = 1200v$$

b) The flux density

$$\Phi = \frac{e\,dt}{dN}$$

Q3.
40kVA
TR 300/20
$E_P = 3300V$

a) EMF secondary winding

Ratio = 300 : 20 max
= 15 20x
So $\frac{3300}{15} = 220V$

b) Primary I + Secondary currents

$I_P = \frac{P}{V}$
$= \frac{40000}{3300}$
$= 12.12A$

$I_S = N \text{ ratio} \times 12.12$
$= 181.8A$

c) max Φ
$E_P = 4.44 N_P f \Phi_{max}$
$3300V = 4.44 \times 300 \times 50Hz \times \Phi$
$\Phi = \frac{3300}{4.44 \times 300 \times 50}$
$= 49.5mWb$

TRANSFORMER PRINCIPLES

Voltage, Current and Flux in an Ideal Iron Cored Coil (purely inductive)

If an alternating voltage V_1 is applied to a pure inductance (no resistance R) and we assume that magnetic saturation does not occur, the current I_1 that flows, will lag the applied voltage V_1 by 90° and will have the same waveshape.

This current produces a flux Φ_1 which is also alternating and in phase with I_1.

The flux Φ_1 induces an emf E_1 into the coil, which is equal and opposite to the applied voltage V_1.

Refer to FIG 1 which shows the circuit diagram and the corresponding phasor diagram.

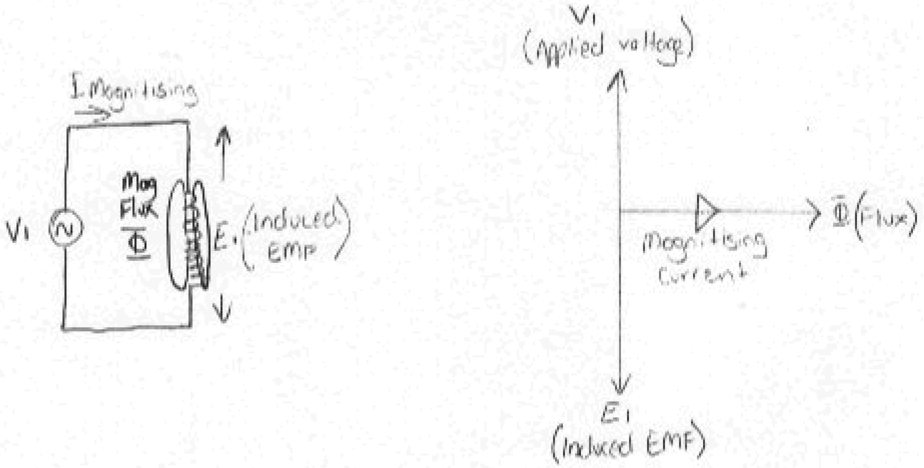

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FIG 1

Notes: Since $V_1 = E_1$, and they are opposite in phase, there is no current flow and hence no flux.

This perfect situation is not possible, and so coils are not ideal but contain a resistive component.

Voltage, Current and Flux in a Practical Coil (has R and L)

In the practical coil, there will be a slight difference between V_1 and E_1 and there will be a current flow, and this current will produce a flux.

The current will cause power losses in the coil resistance (I^2R), and magnetic core losses (hysteresis and eddy current) in the steel core of the coil.

The small resistive component in the coil will cause the current to lag the applied voltage by **less** than 90°.

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Refer to FIG 2 which shows the resulting phasor diagram and equivalent magnetising circuit of the practical coil.

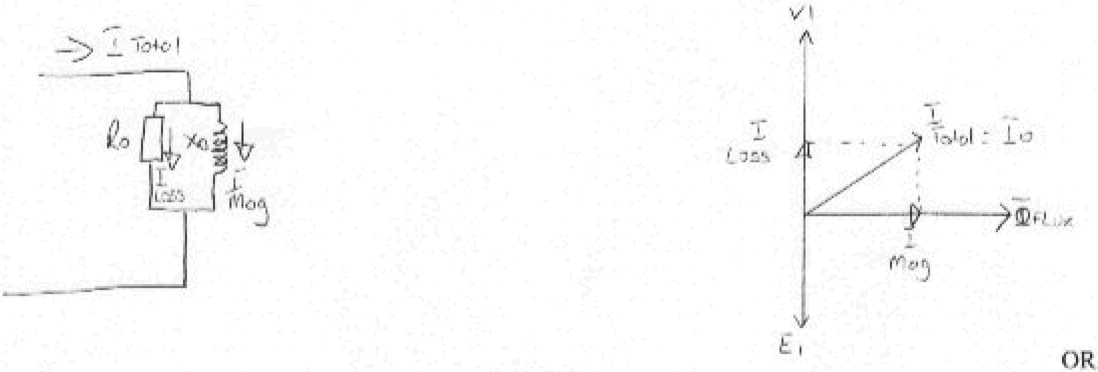

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FIG 2

Notes: The two components shown in the equivalent circuit are in parallel, where R_0 consumes the same power in watts as the iron losses, and X_0 generates the core flux.

The total current I_0 drawn by the coil has two components, I_M and I_L.

The Basic Transformer

A transformer consists of two electrically isolated coils mutually coupled by a common magnetic circuit. Refer to FIG 3 which shows the arrangement of a transformer, source and load.

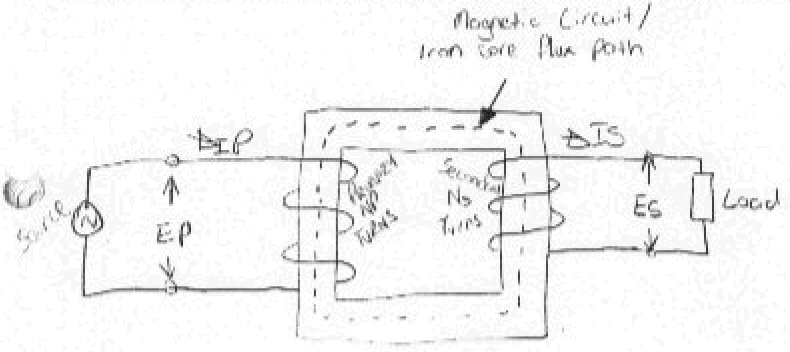

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FIG 3

The "primary" winding with N_P turns is connected to a source of alternating voltage E_P, from which it takes energy, to produce an alternating flux in the magnetic circuit.

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The alternating flux in the magnetic core links the "secondary" winding having N_S turns and induces an alternating voltage E_S into the secondary winding.

The "secondary" winding is now able to supply energy to a connected load.

According to Faraday's Law of Electromagnetic Induction:

Induced emf $\quad e = -N \dfrac{d\Phi}{dt}$ volts

This means that the induced emf in a coil is <u>directly proportional</u> to the number of turns N in the coil.

$$\boxed{\frac{\text{Primary Voltage } E_P}{\text{Secondary Voltage } E_S} = \frac{\text{Primary Turns } N_P}{\text{Secondary Turns } N_S}}$$

If we neglect losses: Power in = Power out

$$E_P I_P = E_S I_S$$

$$\frac{E_P}{E_S} = \frac{I_S}{I_P}$$

Summary:

$$\boxed{\frac{E_P}{E_S} = \frac{I_S}{I_P} = \frac{N_P}{N_S}}$$

$\dfrac{N_P}{N_S}$ is called the "**<u>Turns Ratio</u>**".

Some transformers have more that two windings.

The third winding is called a "tertiary" winding.

Waveshape of Induced Voltage in a Transformer

If a sinewave of voltage is applied to the primary winding of a transformer, then a sinewave of flux will be produced in the magnetic circuit (assuming no saturation effects).

This sinewave of flux will induce a sinewave of voltage into the secondary winding and any other winding, according to the turns ratio.

Sometimes the waveshape of the induced voltage is not perfectly sinusoidal, due to non-linearity in the magnetic circuit and the presence of harmonic components.

EMF Equation for a Transformer

If a transformer is energised by a sinewave of emf then:

Applied Voltage $v = V_{max}\sin\omega t$ volts

and resulting flux $\Phi = \Phi_{max}\sin(\omega t - \pi/2)$ weber.

Assume that there are N_P turns on the primary winding and N_S turns on the secondary winding.

the self-induced emf in the primary winding is:

$$e_P = -N_P\frac{d\Phi}{dt}$$

$$= -N_P\Phi_{max}\frac{d}{dt}(\sin(\omega t - \pi/2))$$

$$= -\omega N_P\Phi_{max}\sin\omega t$$

$$= -E_{Pmax}\sin\omega t$$

This induced voltage is 180° out of phase with the applied voltage v.

So $$E_{Pmax} = \omega N_P\Phi_{max}$$

$$= 2\pi f N_P\Phi_{max}$$

Now $$E_{Prms} = \frac{E_{Pmax}}{\sqrt{2}}$$

$$= \frac{2\pi f N_P\Phi_{max}}{\sqrt{2}}$$

Flux = weber
density = Tesla

This equation simplifies to the equation normally shown in textbooks.

$$\boxed{E_{Prms} = 4.44fN_P\Phi_{max} \quad \text{volts}}$$

Maximum core flux density Bmax is a more useful value to know, to ensure that the core is not saturated.

We can replace Φ_{max} with B_{max} x area of core where a is the cross-sectional area of the magnetic core in m^2.

So the emf equation above can be re-written as:

$$\boxed{E_{Prms} = 4.44fN_PaB_{max} \quad \text{volts}}$$

and for the secondary winding:

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$$\boxed{E_S\text{rms} = 4.44fN_S aB_{max} \quad \text{volts}}$$

Note: The physical area of the magnetic core is usually multiplied by a "**stacking**" factor to allow for the air spaces between the laminations, since the core is not solid.

Example: A transformer has a primary winding of 350 turns and is connected to a 2200Vrms 50Hz sinusoidal supply.
The core length is 125cm, effective core cross-sectional area is 250cm^2, with an air gap of 0.15mm.
Relative permeability μ_R for the steel is 1800.

Calculate:
a) Maximum flux density Bmax in the core
b) RMS magnetising current.

Solution:

Note: Magnetic circuit equations to use are:

Flux density $B = \mu_0\mu_R H$

Magnetising force $H = NI/l$

$\mu_0 = 4\pi x 10^{-7}$

a)

$$E_P = 4.44fN_P aB_{max}$$

$$B_{max} = \frac{E_P}{4.44fN_P a}$$

$$= \frac{2200}{4.44x50x350x250x10^{-4}}$$

$$= 1.13 \text{ Tesla}$$

b) Now $B_{max} = \mu_0\mu_R H_{max}$

For the steel core:

$$H_{max} = \frac{B_{max}}{\mu_0\mu_R}$$

$$= \frac{1.13}{4\pi x10^{-7}x1800}$$

$$= 500 \text{ AT/metre}$$

MMF $= NI = Hl = 500x1.25 = 625AT$

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For the air gap:

$$H_{max} = \frac{B_{max}}{\mu_0}$$

$$= \frac{1.13}{4\pi x 10^{-7}}$$

$$= 89922 \text{ AT/metre}$$

$$\text{MMF} = NI = Hl = \frac{1.13x1.5x10^{-4}}{4\pi x10^{-7}}$$

$$= 135 \text{ AT}$$

$$\text{Total MMF} = 625 + 135 = 760 \text{ AT}$$

Magnetising current in Primary winding $I_{Pmax} = \frac{NI_{max}}{N}$

$$I_{Pmax} = \frac{760}{350} = 2.17A$$

$$Irms = 2.17x0.707 = 1.53A$$

Losses in a Transformer

There are two types of energy losses in a transformer:

a) "Iron" losses (magnetic circuit losses)

Power is lost in magnetising the steel core of the transformer.

Hysteresis loss is the energy required to magnetise and de-magnetise the core for each cycle of the alternating flux.

This loss is reduced by reducing the weight of steel in the core, since **Hysteresis Loss units are Watts/Kg of core steel/cycle of supply**

Eddy Current loss is the power dissipated by induced currents circulating in the steel core.

This loss is reduced by building the magnetic core with thin laminations instead of a solid steel core.

The laminations are 0.35mm thick and are coated with an electrical insulating layer, to increase the electrical resistance of the core.

This decreases the circulating currents and hence the power loss and the laminations are clamped together so that they still provide a good path for the flux.

b) "copper" losses (winding losses) (sometimes called "load" losses)

The primary and secondary windings of the transformer have resistances R_P and R_S respectively, and when current passes through the windings there is a power loss (I^2R).

Power Loss in Primary = $I_P{}^2R_P$ watts

Power Loss in Secondary = $I_S{}^2R_S$ watts

These power losses are proportional to (current)^2, and will vary as the load varies. This means that the copper loss at ½ load will be ¼ of the loss at full load.

Effects of Losses in Transformers

Iron losses result in current being drawn from the source by the primary winding to magnetise the core, even if no load is connected to the secondary.

This current is called "magnetising" or "no load" current.

This means that when the transformer is loaded, the primary current will be slightly greater than expected from the ratio $\frac{I_P}{I_S} = \propto \frac{N_S}{N_P}$.

Copper losses or winding resistance cause voltage drops in the windings and this will cause the secondary voltage to be less than expected from the ratio $\frac{E_P}{E_S} = \frac{N_P}{N_S}$ when the transformer is loaded.

Internal voltage drops across R_P and R_S cause the secondary voltage to drop.

Ideal Transformer

An "ideal" transformer has no losses and has perfect transformation according to turns ratio.

The "ideal" transformer is used in many calculations to approximate conditions because most "practical" transformers are >95% efficient.

Practical Transformer

The "practical" transformer has losses such as iron and copper losses.

The loss components in a practical transformer can be represented in a transformer "equivalent circuit".

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Equivalent Circuit of a Transformer

Refer to FIG 5 which is an equivalent circuit of a transformer.

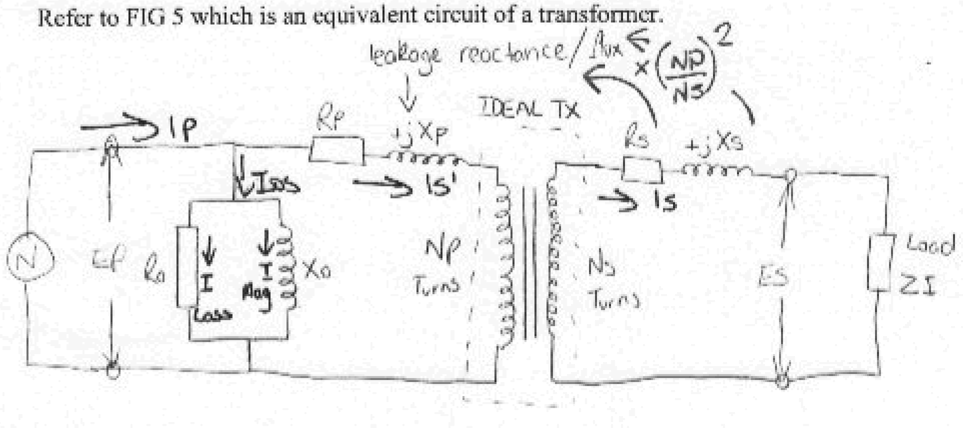

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FIG 5

The equivalent circuit is drawn to represent all losses in the transformer by conventional circuit components (R and X_L), and allows analysis of the transformer performance under varying load conditions.

R_P	Primary winding resistance
R_S	Secondary winding resistance

R_P and R_S will cause power losses (copper losses) when the transformer is loaded ($I_P{}^2R_P$ in primary and $I_S{}^2R_S$ in secondary)

R_P and R_S will also cause voltage drops (I_PR_P and I_SR_S).

X_P	Primary Leakage Reactance
X_S	Secondary Leakage Reactance

Leakage Flux and Leakage Reactance

In a practical transformer, there will not be perfect magnetic flux linkage between the primary and secondary windings.

Not all of the primary flux will link the secondary winding, and not all of the secondary flux will link the primary winding.

Higher leakage lower enduced voltage

$\therefore I \propto EMF$

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Leakage reactance represents lost flux in the transformer, between primary and secondary windings and results in lower induced voltage but no loss of power.

X_P and X_S are connected in series, so that they will cause voltage drops (I_PX_P and I_SX_S) representing lost flux in the core.

The primary winding is represented by: $R_P + jX_P$
The secondary winding is represented by: $R_S + jX_S$

R_0	Consumes same power as total iron losses (hysteresis and eddy current) ($E_P{}^2/R_0$).
X_0	Reactor to produce core flux.

Now that all of the losses in the transformer are represented by circuit components, there remains an "ideal" transformer with perfect transformation, and this is shown as the link between the primary and secondary of the transformer.

When the transformer is loaded, currents will cause voltage drops in R_P, R_S, X_P and X_S so that the secondary terminal voltage will drop.

The no load or magnetising current I_0 is the total current that flows into the parallel circuit of R_0 and X_0.

Equivalent Circuit used to determine Transformer Performance

The equivalent circuit is used to determine transformer performance under different load conditions.

All of the loss components are included and if the supply voltage and load conditions are known, then voltages, currents and power values on both sides of the transformer can be calculated.

The equivalent circuit can be simplified by referring all quantities to one side of the transformer.

Referred values are identified by using superscript notation.

Example: Secondary resistance R_S when referred to the primary side is written as:

$$R_S' = R_S \times (N_P/N_S)^2\ \Omega$$

Similarly, primary resistance R_P when referred to the secondary side is written as:

$$R_P' = R_P \times (N_S/N_P)^2\ \Omega$$

Note: Voltages and currents are referred by using the turns ratio.

I and V $\left(\frac{NS}{NP}\right)$

R and XL $\left(\frac{NS}{NP}\right)^2$

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Simplified Equivalent Circuit (Referred to Primary)

All components, voltages and currents on the secondary side of the transformer (including the load) can be moved to the primary side and replaced by their equivalent referred values.

FIG 6 shows the equivalent circuit simplified by referring all secondary quantities to the primary side.

$R_S' = R_S \times \left(\frac{N_P}{N_S}\right)^2$

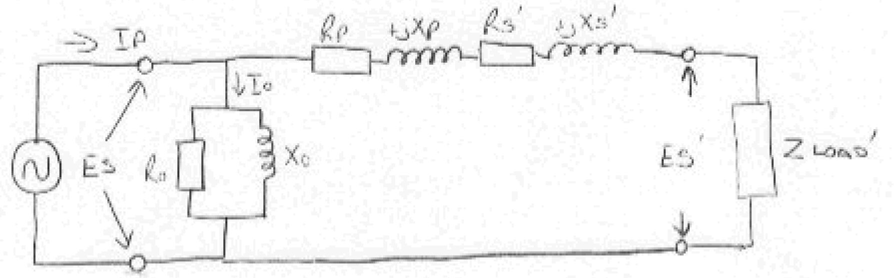


FIG 6

Note: This simplification has eliminated the ideal transformer, and the whole circuit (both primary and secondary) is now represented by a series/parallel circuit which can be easily solved if the values of R_P, R_S, X_P, X_S, R_0, X_0 and load impedance Z_L are known in complex form.

One further simplification can be made to the equivalent circuit by combining the components to give:

$Rep = (R_P + R_S')$ Total equivalent resistance referred to primary

$Xep = (X_P + X_S')$

This simplified circuit is shown in FIG 7.

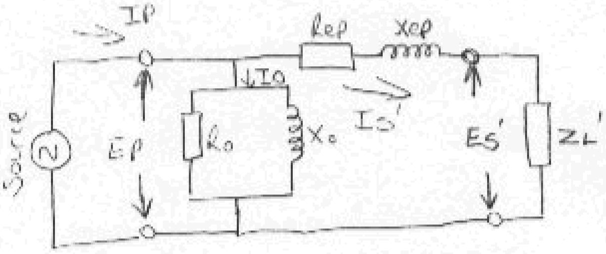
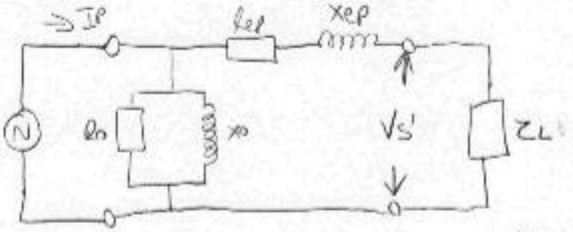


FIG 7

Example: Refer to FIG 8 which shows the simplified equivalent circuit of a transformer with all quantities referred to the primary side and the primary supply voltage is 200Vrms. Turns ratio is 1:10



$R_{ep} = 0.16\Omega$
$X_{ep} = +j\,0.7\Omega$
$R_0 = 400\Omega$
$X_0 = +j\,231\Omega$
$V = 200v\angle 0°$
$Z_L = 5.96 + j\,4.44$

$\frac{N_P}{N_S} = \frac{1}{10}$ step-up tx

FIG 8

Calculate:

a) iron losses,

b) no-load current,

c) secondary load current,

d) total primary current,

e) copper losses,

f) Secondary terminal voltage V_{load},

g) Power in watts consumed by the load

h) percent efficiency.

Solution:

Assume supply voltage V_P is reference ($200\underline{/0°}$ volts)

R_0: Loss total

a) Iron Losses $= V_P{}^2/R_0 = 200^2/400 = 100W$

b) No load current $I_0 = (I_{R0} + I_{X0})$

$= (V_P/R_0)\underline{/0°} + (V_P/X_0)\underline{/-90°}$

$= (200/400)\underline{/0°} + (200/231)\underline{/-90°}$

$= 0.5 - j0.86$

$= 1\underline{/-59.8°}$ amps

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c) Secondary Load current referred to Primary

$$I_S' = V_P/(Zep + Z_L')$$

$$= \frac{200\underline{/0^\circ}}{(0.16 + j0.7 + 5.96 + j4.44)}$$

$$= \frac{200\underline{/0^\circ}}{7.9\underline{/39.2^\circ}}$$

$$= 25.3\underline{/-39.2^\circ} \text{ amps}$$

Secondary load current:

$$I_S = I_S' \times N_P/N_S$$

$$= 25.3 \times 1/10$$

$$= 2.53\underline{/-39.2^\circ} \text{ amps}$$

d) Total Primary Current:

$$I_P = (I_0 + I_S')$$

$$= (1\underline{/-59.8^\circ} + 25.3\underline{/-39.2^\circ})$$

$$= (0.5 - j0.86) + (19.6 - j16)$$

$$= 20.1 - j16.86$$

$$= 26.2\underline{/-40^\circ} \text{ amps}$$

e) Copper losses:

$$P_{copper} = (I_S')^2 Rep$$

$$= (25.3)^2 x0.16$$

$$= 102.4W$$

f) Secondary Terminal Voltage Referred to Primary:

$$V_S' = I_S' x Z_L'$$

$$= 25.3\underline{/-39.2^\circ} \times (5.96 + j4.44)$$

$$= \overset{\times}{25.3}\underline{/\overset{+}{-39.2^\circ}} \times \overset{\times}{7.43}\underline{/\overset{+}{36.7^\circ}}$$

$$= 188\underline{/-2.5^\circ} \text{ volts}$$

Secondary Terminal Voltage

$$V_S = V_S' \times N_S/N_P$$

$$= 188\underline{/-2.5^\circ} \times 10/1$$

$$= 1880\underline{/-2.5^\circ} \text{ volts}$$

g) Load Power

$$P_L = I_S'^2 R_L'$$

$$= (25.3)^2 x5.96$$

$$= 3815\text{W}$$

h) Efficiency %

$$\text{Eff \%} = \frac{\text{Pout x 100}}{(\text{Pout + Plosses})}$$

$$= \frac{3815 \text{ x } 100}{3815 + \underset{\text{Iron}}{100} + \underset{\text{Copper}}{102.4}}$$

$$= 95\%$$

Tests

No load $\Rightarrow$ R_o and X_o

Short circuit $\Rightarrow$ R_{ep} and X_{ep}

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TRANSFORMER TESTING

Equivalent Circuit Determined by Testing

Phasor Diagram of Transformer on No Load

Refer to FIG 1 which shows the simplified equivalent circuit of a transformer with all quantities referred to the primary side.

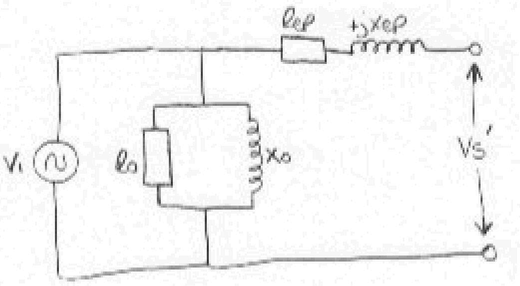

FIG 1

If a test voltage V_1 is applied to the primary winding with the secondary winding left open circuited (no load connected) the resulting phasor diagram will be as shown in FIG 2.

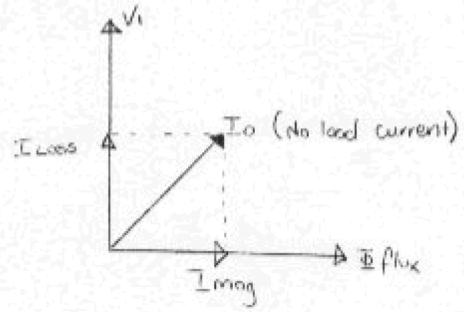

FIG 2

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No Load or Open Circuit Test to Determine Iron Losses

Normal "**rated**" voltage is applied to one winding of a transformer with the other winding left open circuited as indicated above.

The test circuit is as shown in FIG 3.

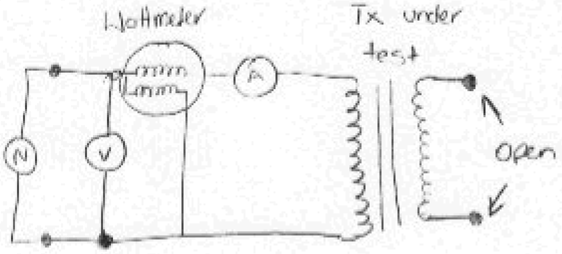


FIG 3

The quantities measured are test voltage V_0 in volts, test current I_0 in amps and power consumed P_0 in watts which are the magnetising circuit losses.

From the values of V_0, I_0 and P_0, we can determine the values of R_0 and X_0 in the magnetising equivalent circuit.

Example: A no load test carried out on a 200/400 volt, 4kVA 50Hz power transformer gave the following results.

$V_0 = 200V$ $I_0 = 0.7A$ $P_0 = 60$ W (iron losses)

Determine the values of R_0 and X_0 and draw the equivalent magnetising circuit.

Solution:

P_0 = power lost in R_0

$$P_0 = V_0^2/R_0$$

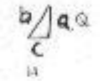
$$R_0 = V_0^2/P_0 = 200^2/60 = = 666.7\Omega$$

To determine X_0, we must first calculate the reactive power in VARS consumed by X_0.

Apparent power in circuit $= V_0 x I_0 =$ 200x0.7 $=$ 140VA

$a^2 = b^2 - c^2$

Reactive power in VARS $= \sqrt{(VA^2 - WATTS^2)} = \sqrt{(140^2 - 60^2)}$



$= 126.5$ VARS

$X_0 = V_0^2/VARS = 200^2/126.5 = 316\Omega$

Short Circuit Test to Determine Copper Losses

A short circuit is applied to one side of a transformer while the voltage applied to the other side of the transformer is gradually increased until "**rated**" current is flowing in the short circuit.

The test circuit is as shown in FIG 4.

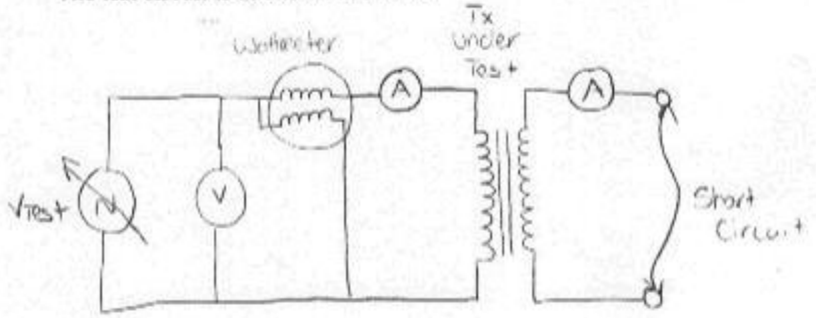


FIG 4

The quantities measured are test voltage V_{SC} in volts, current flowing in the short circuit I_{SC} in amps and power consumed P_{SC} in watts which are the total copper losses.

From the values of V_{SC}, I_{SC} and P_{SC}, we can determine the values of Req and Xeq in the equivalent circuit by using the equations:

$$Zeq = V_{SC}/I_{SC} \quad \text{and} \quad P_{SC} = I_{SC}^2/Req$$

Notes: The test can be carried out on **either** side of the transformer.

When the test voltage is applied to the primary side with the secondary side shorted, values of Rep and Xep are determined.

When the test voltage is applied to the secondary side with the primary side shorted, values of Res and Xes are determined.

The voltage required for the test is much **less** than rated voltage.

DO NOT APPLY FULL RATED VOLTAGE otherwise the transformer will be damaged by excessive current flow.

As the test voltage is very small, it is assumed that negligible current I_0 flows through the magnetisation circuit.

Example: A short circuit test is applied to a 200/400V 4kVA 50Hz transformer with test voltage applied to the 400V side, and the short circuit applied to the 200V side of the transformer.

The test results were:

$V_{SC} = 9V$ $\quad$ $I_{SC} = 6A$ $\quad$ $P_{SC} = 21.6W$ (copper losses)

Determine the values of Rep and Xep and draw the equivalent circuit with quantities referred to the primary (200V) side.

Solution:

P_{SC} is dissipated in Res since the test voltage is applied to the secondary side of the transformer.

$P_{SC} \quad = \quad I_{SC}^2/Res$

$Res \quad = \quad P_{SC}/I_{SC}^2 \quad = \quad 21.6/6^2 = \quad 0.6\Omega$

$Zes \quad = \quad V_{SC}/I_{SC} \quad = \quad 9/6 \quad = \quad 1.5\Omega$

$Xes \quad = \quad \sqrt{(Zes^2 - Res^2)}$

$\quad = \quad \sqrt{(1.5^2 - 0.6^2)} \quad = \quad 1.37\Omega$

Transfer quantities to the primary (LV) side.

$Rep \quad = \quad Res \times (N_P/N_S)^2 = \quad 0.6 \times (200/400)^2 \quad = \quad 0.15\Omega$

$Xep \quad = \quad Xes \times (N_P/N_S)^2 \quad = \quad 1.37 \times (200/400)^2 \quad = \quad 0.34\Omega$

When the test voltage is applied to the secondary side with the primary side shorted, values of Res and Xes are determined.

The voltage required for the test is much **less** than rated voltage.

DO NOT APPLY FULL RATED VOLTAGE otherwise the transformer will be damaged by excessive current flow.

As the test voltage is very small, it is assumed that negligible current I_0 flows through the magnetisation circuit.

Example: A short circuit test is applied to a 200/400V 4kVA 50Hz transformer with test voltage applied to the 400V side, and the short circuit applied to the 200V side of the transformer.

The test results were:

$V_{SC} = 9V$ $I_{SC} = 6A$ $P_{SC} = 21.6W$ (copper losses)

Determine the values of Rep and Xep and draw the equivalent circuit with quantities referred to the primary (200V) side.

Solution:

P_{SC} is dissipated in Res since the test voltage is applied to the secondary side of the transformer.

$P_{SC} = I_{SC}^2/Res$

$Res = P_{SC}/I_{SC}^2 = 21.6/6^2 = 0.6\Omega$

$Zes = V_{SC}/I_{SC} = 9/6 = 1.5\Omega$

$Xes = \sqrt{(Zes^2 - Res^2)}$

$= \sqrt{(1.5^2 - 0.6^2)} = 1.37\Omega$

Transfer quantities to the primary (LV) side.

$Rep = Res \times (N_P/N_S)^2 = 0.6 \times (200/400)^2 = 0.15\Omega$

$Xep = Xes \times (N_P/N_S)^2 = 1.37 \times (200/400)^2 = 0.34\Omega$

TR35

From the two tests carried out above, the total equivalent circuit referred to the LV side can be drawn. as shown in FIG 5.

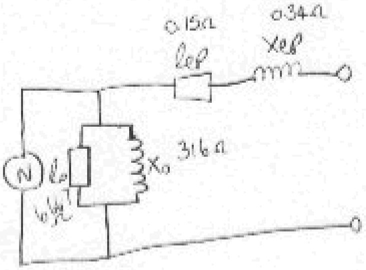
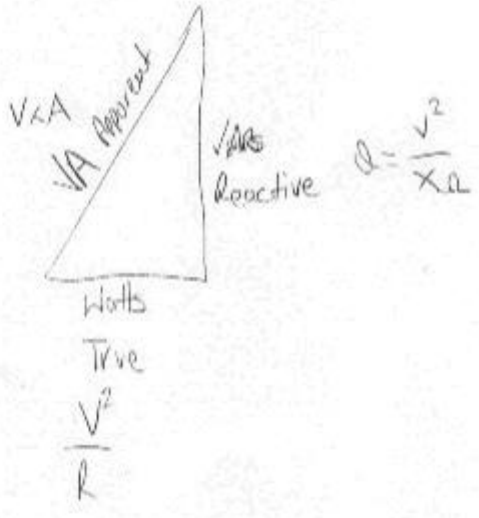


FIG 5

R_o = Iron Watts

X_o : Reactances Loss Flux

$R_{ep} + jX_{ep}$: Copper Losses



TR37

Tut 3

a. Voltage Regulation : is the variation of the secondary voltage between no load and full load, expressed as a percentage of the no load voltage, assuming the primary is constant.

c. %Rd = 0.9%   p
%Xd = 5.4%

i. $\%Zd = R\% \cos\theta + X\% \sin\theta$ pf = 0.8 $\cos^{-1} 0.8 = 36.87$
$= 0.9 \times 0.8 + 5.4 \times 0.6$
$= 3.96\%$ $\sin 36.87 = 0.6$

ii. max regulation p.f $\frac{X\%}{R\%} = \frac{5.4}{0.9} = 6$ ∴ tan 6 = 0.105

a. A short circuit test will give you

b. 100kVA
6600/250
Shorted : 10A, 450W, 100V p

i. V_p =, when secondary = 250v full load pf 0.8 lag

$V_s = \frac{450}{10} = 46V$

∴ Ratio = 100.45
= 2.22

3. $R\% = 2$

$X\% = 4$

0.8 p.f

$Reg\ \% = R\% \cos\theta_2 + X\% \cos\theta_2$

$= 2 \times 0.8 + 4 \times 0.6$

$= 4\%$

4. 500 kVA

2500/415V 50Hz

$R_0 = 750\Omega$ $X_0 = +j2150\Omega$

$R_{EP} = 0.1875\Omega$ $X_{EP} = +j0.8125\Omega$

a) $R\% = \frac{I_s' \times R_{ep}}{V_p} \times \frac{100}{1}$

$= 1.5\%$

$I_s = \frac{500000}{2500} = 200$

$X\% = \frac{I_s' \times X_{ep}}{V_p} \times 100$

$= 6.5\%$

$Z_{ep} = R_{ep} + jX_{ep}$

$= 0.1875 + j0.8125$

$= 0.833 \angle 77$

$\%Z\% = \frac{I_s' Z_{ep}}{V_p} = 6.67\%$

b) secondary terminal voltage

500kVA 0.6 pf leading

5. 3300/660V

$R_p = 0.8\Omega$

$R_s = 0.03\Omega$

$X_p = j4\Omega$

$X_s = j0.12\Omega$

TRANSFORMER VOLTAGE REGULATION

Voltage Regulation of a transformer is the variation of the secondary voltage between no load and full load, expressed as a percentage of the no load voltage, assuming that the primary voltage is constant.

$$\boxed{\text{\% Regulation} = \frac{(\text{No-load Voltage - Full Load Voltage}) \times 100}{\text{No-load Voltage}}}$$

Refer to the transformer equivalent circuit shown in FIG 1.

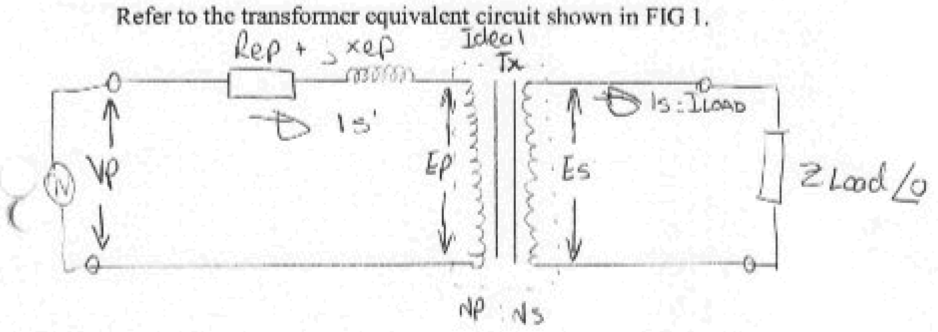


FIG 1

Note: The magnetisation circuit has not been included because it does not affect voltage regulation.

Voltage regulation is caused by the voltage drop across Zep which occurs when primary load current Is' passes through it.

On no-load, there will be no current flowing, and so no voltage drop through the transformer.

Refer to FIG 2 which shows the phasor diagram for the transformer in FIG 1.

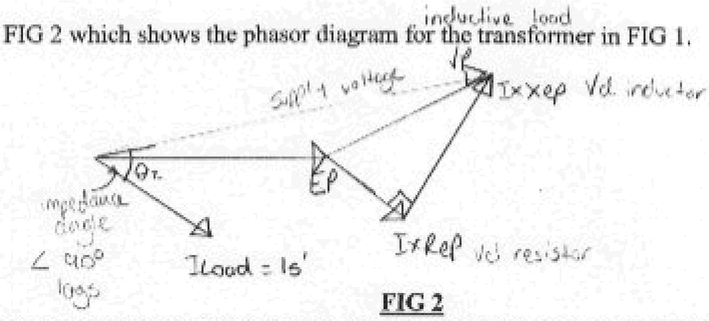


FIG 2

The difference between Ep and Vp is called "**Regulation**", but we must consider the magnitude and phase difference between the two voltages.

VP Supply voltage
Ep = Supply voltage - Losses

TR41

It can be shown that:
For lagging power factor loads, Add Losses - inductive loads

$$\boxed{\%\,\text{Regulation} = \frac{Is'(Rep\cos\theta_2 + Xep\sin\theta_2) \times 100}{Vp}}$$

For leading power factor loads, minus Losses - capacitive load

$$\boxed{\%\,\text{Regulation} = \frac{Is'(Rep\cos\theta_2 - Xep\sin\theta_2) \times 100}{Vp}}$$

Note: These equations can be re-written and calculated using secondary side values.

Percentage Equivalent Impedance Z%, Resistance R% and Reactance X%

Z% is defined as the IZ voltage drop across the windings when **rated** current is passing through them, expressed as a percentage of the **rated** voltage Vp.

$$\boxed{Z\% = \frac{Is' \times Zep \times 100}{Vp}}$$

Percentage of voltage drop

Similarly the voltage drop across the total resistance of a transformer is called the resistance voltage drop and can be written as a percentage of the **rated** voltage Vp.

$$\boxed{R\% = \frac{Is' \times Rep \times 100}{Vp}}$$

The voltage drop across the total leakage reactance of a transformer is called the reactance voltage drop and can also be written as a percentage of the **rated** voltage Vp.

$$\boxed{X\% = \frac{Is' \times Xep \times 100}{Vp}}$$

These voltage drops are expressed as a percentage of primary applied voltage Vp (100%).

If expressed as per unit voltages, then they are referred to the primary applied voltage Vp (1 pu).
Substituting Z%, R% and X% in the regulation equation and assuming Vp = 100%:

$$\boxed{\text{Regulation}\,\% = R\%\cos\theta_2 + X\%\sin\theta_2}$$

$Zep = Rep + jXep$

$\theta_2 =$ ← = Refer to Secondary winding

TR42

Example: A transformer has percentage equivalent resistance (Rep) of 2%, and percentage leakage reactance of 4%. Xep

Calculate the voltage regulation when the transformer is supplying full rated load at 0.8 power factor lagging.

log load so + Rep and Xep

$$\% \text{ Regulation} = \frac{\text{Is}'(\text{Rep}\cos\theta_2 + \text{Xep}\sin\theta_2) \times 100}{\text{Vp}}$$

Also

$$\% \text{ Regulation} = \text{R}\%\cos\theta_2 + \text{X}\%\sin\theta_2$$

$$= (2\% \text{x } 0.8) + (4\% \text{x } 0.6)$$

$$= 4\%$$

Example: A 100kVA transformer has 400 turns on the primary winding and 80 turns on the secondary winding.

The primary resistance Rp is 0.3Ω, the secondary resistance Rs is 0.01Ω, the primary leakage reactance Xp is 1.1Ω and secondary leakage reactance is 0.035Ω.

The supply voltage is 2200V and secondary ratio voltage is 440V.

Calculate:

a) the equivalent impedance referred to the primary,

b) voltage regulation and secondary terminal voltage for

i) full load 0.8 pf lagging,

ii) full load 0.8 pf leading.

400 : 80
5

Solution:

a)

$$\text{Rep} = \text{Rp} + \text{Rs}'$$

$$= 0.3 + \boxed{0.01(400/80)^2}$$ secondary to primary

$$= 0.55\Omega$$

$$\text{Xep} = \text{Xp} + \text{Xs}'$$

$$= 1.1 + 0.035(400/80)^2$$

$$= 1.975\Omega$$

$$\text{Zep} = 0.55 + j1.975$$

$$= 2.05\underline{/74.4°}\Omega$$

TR43

b) Draw the equivalent circuit as shown in FIG 3.

$R + jX_L = \angle\theta \quad - I\angle\theta$

$R - jX_C = \angle-\theta \quad + I\angle\theta$

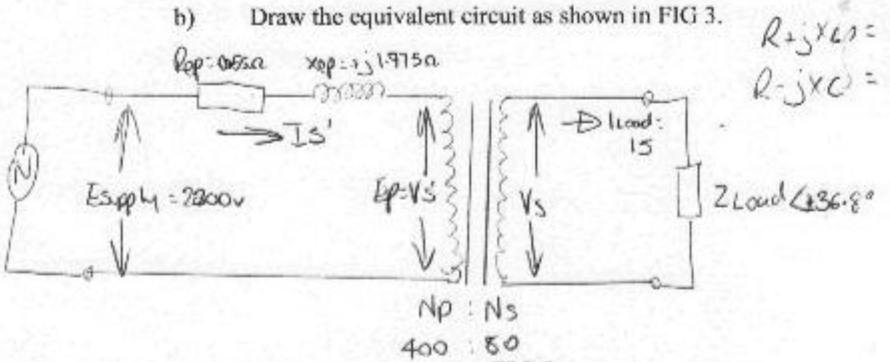


FIG 3

i) $I_s = I_{load} = \dfrac{\underline{VArated}}{Vsrated}$

$= \dfrac{\underline{100 \times 10^3}}{440}$ 100kVA

$= 227.3\underline{/-36.9^\circ}$A 0.8

$I_s' = 227.3 \times (Ns/Np)$

$= 45.45\underline{/-36.9^\circ}$A

Voltage drop across Zep $= I_s' \times Zep$

$= 45.45\underline{/-36.9^\circ} \times 2.05\underline{/74.4^\circ}$

$= 93.2\underline{/37.5^\circ}$ Volts

$V_s' = Vp - I_s'xZep$

$= 2200\underline{/0^\circ} - 93.2\underline{/37.5^\circ}$

$= (2200 + j0) - (73.9 + j56.7)$

$= 2126.1 - j56.7$

$= 2126.8\underline{/-1.5^\circ}$

% Regulation $= \dfrac{\underline{(2200 - 2126.8) \times 100}}{2200}$

$= 3.33\%$

Can also be solved using the simplified formula:

$$\% \text{ Regulation} = \frac{Is' (Repcos\theta_2 + Xepsin\theta_2) \times 100}{Vp}$$

$$= \frac{45.45(0.55 \times 0.8 + 1.975 \times 0.6) \times 100}{2200}$$

$$= 3.34\%$$

$$\text{Secondary terminal voltage} = 440 \times (100 - 3.34)$$

$$= 425.2V$$

ii) For 0.8 leading power factor

$$\% \text{ Regulation} = \frac{Is'(Repcos\theta_2 - Xepsin\theta_2) \times 100}{Vp}$$

$$= \frac{45.45(0.55 \times 0.8 - 1.975 \times 0.6) \times 100}{2200}$$

$$= - 1.54\%$$

Note: The negative sign for regulation indicates that there is a rise in secondary voltage due to the capacitive load.
This means that the secondary terminal voltage is higher than the ratio voltage.

$$\text{Secondary terminal voltage} = 440 \times (100 + 1.54)$$

$$= 446.8V \text{ (ratio voltage is 440)}$$

TR45

Worst case senorio

Maximum Voltage Regulation of a Transformer

The phasor diagram shown in FIG 1 indicates that as the power factor of the load (θ_2)changes the triangle of Is'Rep, Is'Xep and Is'Zep rotates around the end of Ep.

Refer to FIG 4 which shows a phasor diagram when Ep and Vp are in phase.

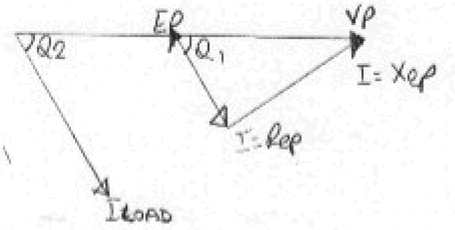


FIG 4

Maximum voltage regulation occurs when there is the greatest difference between Ep and Vp.

$$\% \text{ Regulation} = \frac{(Vp - Ep) \times 100}{Vp}$$

Using the regulation equation:

$$\% \text{ Regulation} = \frac{Is'(Rep\cos\theta_2 + Xep\sin\theta_2) \times 100}{Vp}$$

$$= (R\% \times \cos\theta_2) + (X\% \times \sin\theta_2)$$

Load power factor angle (0_2) when Maximum Voltage Regulation Occurs

Maximum regulation occurs when $\frac{dReg\%}{d\theta_2} = 0$

$$dReg\% = R\%(-\sin\theta_2) + X\%(\cos 0_2) = 0$$

$$X\%\cos 0_2 = R\%\sin\theta_2$$

$$\frac{X\%}{R\%} = \frac{\sin\theta_2}{\cos 0_2}$$

$$X\%/R\% = \tan 0_2$$

TR46

Max Regulation occurs when θ_2 = tan-1(Xeq/Req)

Best case senario

Minimum Voltage Regulation of a Transformer Zero voltage drop, leading PF

Refer to FIG 5 which shows a phasor diagram when Ep and Vp are equal.

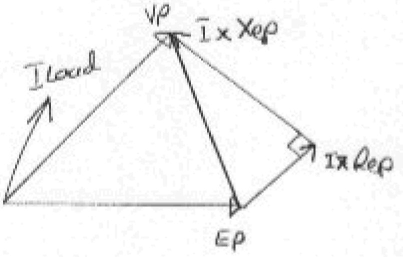


FIG 5

Minimum (zero) voltage regulation occurs when Ep and Vp are equal.

$$\% \text{ Regulation} = \frac{(Vp - Ep) \times 100}{Vp} = 0$$

Load power factor angle (θ_2) when Minimum Voltage Regulation Occurs

$$\text{Reg\%} = R\%(\cos\theta_2) + X\%(\sin\theta_2) = 0$$

$$X\%\sin\theta_2 = - R\%\cos\theta_2$$

$$\frac{\sin\theta_2}{\cos\theta_2} = \frac{- R\%}{X\%}$$

$$\tan\theta_2 = - Req/Xeq$$

$$\theta_2 = \tan\text{-}1(-Req/Xeq)$$

Zero Regulation occurs when θ_2 = tan-1(-Req/Xeq)

TRANSFORMER LOSSES AND EFFICIENCY

Iron Losses

Hysteresis and Eddy Current losses in a transformer are represented by the power consumed by R_0 in the equivalent circuit, and is measured during the Open Circuit (No Load) Test as P_{OC}.

The no load losses depend on transformer supply voltage but are independent of load current.

$$\text{Iron Loss} = P_{OC} = \frac{\text{Vsupply}^2}{R_0}$$

Copper Losses

Winding power losses in a transformer are represented by the power consumed by primary resistance Rp and secondary resistance Rs in the equivalent circuit, and is measured during the Short Circuit Test as P_{SC}.

The copper losses depend on the value of load current drawn from the transformer.

$$\text{Copper Loss} = P_{SC} = \text{Is}^2\text{Res} \quad \text{OR} \quad (\text{Is}')^2\text{Rep}$$

Efficiency of a Transformer

$$\text{Efficiency \%} = \frac{\text{Load Power in Watts}}{\text{Total Input Power}} \text{x100}$$

$$= \frac{\text{Iload}^2\text{Rload}}{\text{Iload}^2\text{Rload} + \text{Plosses}} \text{x100}$$

$$\text{Efficiency \%} = \frac{I_L^2 R_L}{I_L^2 R_L + \underset{\uparrow \text{ Open circuit}}{P_{OC}} + \underset{\nwarrow \text{ Short circuit}}{P_{SC}}} \text{x100}$$

Can also be written as:

$$\text{Full Load Efficiency \%} = \frac{\text{Full Load VA x Power Factor x 100}}{(\text{Full Load VA x Power Factor}) + P_{OC} + P_{SC}}$$

TR48

For any other fraction of Full Load where n = 0 to 1
(**Example**: for 50% full load $n = 0.5$)

$$\boxed{\text{Efficiency \%} = \frac{n \times \text{Full Load VA} \times \text{Power Factor} \times 100}{n \times (\text{Full Load VA} \times \text{Power Factor}) + P_{OC} + n^2 P_{SC}}}$$

Example: A 500kVA transformer has a voltage ratio of 6600/400V, iron losses of 2.9kW and total **full load** copper losses of 4kW.
Calculate:
- i) efficiency % at full load 0.8 pf lagging
- ii) efficiency % at half full load 0.8 pf lagging.

Copper loss = 1kw
! better!

Solution:

i) Full Load Eff %

$$= \frac{\text{Full Load VA} \times \text{Power Factor} \times 100}{(\text{Full Load VA} \times \text{Power Factor}) + \underset{\text{Iron}}{P_{OC}} + \underset{\text{Copper}}{P_{SC}}}$$

$$= \frac{500 \times 10^3 \times 0.8 \times 100}{(500 \times 10^3 \times 0.8) + (2.9 \times 10^3) + (4 \times 10^3)}$$

$$= 98.3\%$$

ii) ½ Full Load Eff %

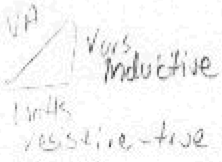
$$= \frac{n \times \text{Full Load VA} \times \text{Power Factor} \times 100}{n \times (\text{Full Load VA} \times \text{Power Factor}) + P_{OC} + n^2 P_{SC}}$$

$$= \frac{0.5 \times 500 \times 10^3 \times 0.8 \times 100}{0.5 \times (500 \times 10^3 \times 0.8) + (2.9 \times 10^3) + (0.5)^2 \times (4 \times 10^3)}$$

$$= 98.1\%$$

Note: Iron losses are constant but copper losses are proportional to the (load current)^2.

Copper losses vary with the load



Tr 49

Maximum Efficiency of a Transformer

It can be shown that a transformer has maximum efficiency when Iron Losses have the same value as Copper Losses

This will occur at a particular fraction of full load "n".

Losses

Iron = Copper

At Maximum Efficiency $P_{OC} = P_{SC}$

Also:

At Maximum Efficiency $P_{OC} = n^2 x P_{SC}$(full load)

Load Fraction "n" at which Maximum Efficiency Occurs

Re-arranging the maximum efficiency equation above:

At Maximum Efficiency $n = \sqrt{(P_{OC}/P_{SCFL})}$

Referring to the example above:

Calculate: iii) the load at which maximum efficiency occurs,
iv) the value of maximum efficiency.

Solution:

$(n \text{ factor}^2)$

iii) Load Fraction n $= \sqrt{(P_{OC}/P_{SCFL})}$

$= \sqrt{(2.9x10^3/4x10^3)}$

$= 0.851$ (85.1% of Full Load)

Note: This corresponds to: 85.1% of 500kVA = 425.5kVA
or 85.1% of 400kW = 340.6kW (at 0.8 pf lag)

iv) Maximum Eff % $= \dfrac{n \text{ x Full Load VA x Power Factor x } 100}{n \text{ x (Full Load VA x Power Factor)} + P_{OC} + n^2 P_{SC}}$

$= \dfrac{0.851x500x103 \text{ x } 0.8 \text{ x } 100}{(0.851x500x10^3x0.8) + (2.9x10^3) + (2.9x10^3)}$

$= 98.33\%$

max efficiency

Note: If the values of Rep, Xep, R_0 and X_0 are known, then P_{OC} and P_{SC} may need to be calculated.

Poc + Psc are =

TR50

All Day Efficiency of a Transformer

A power transformer may be connected to the supply and energised, for long periods of time.

There may be varying levels of load on the transformer, for different periods of time.

Sometimes the transformer may be energised but unloaded.

Efficiency will vary, depending on the level of load.

Full load efficiency is not the only criterion to consider when selecting a suitable transformer.

We must consider losses at full load, fractional load and at no load over a 24 hour period, and this is called "**All Day Efficiency**".

All Day Efficiency is the ratio of (Energy Output/Energy Input) of the transformer over a given period (usually 24 hours).

Note: The units of Energy are Kilowatt Hours (kWh).

$$\boxed{\text{All Day Efficiency \%} = \frac{\text{Energy output for 24 Hrs} \times 100}{\text{Energy Input for 24 Hrs}} = \frac{\text{kWh out}}{\text{kWh in}}}$$

Energy Output in kWh = Power Output in kW x Time in Hours

Energy Input in kWh = Power Input in kW x Time in Hours

= (Power Out + Losses) x Time in Hours

Example: A 100kVA single phase power transformer has iron losses of 500W and full load copper losses of 750W.

The transformer has a 24 hour load cycle as follows:

8 hours at 80kW 0.8 power factor lagging,
6 hours at 50kVA 0.9 power factor lagging,
4 hours at 25 kVA and 20kW,
3 hours energised but no load,
3 hours de-energised.

Calculate the all day efficiency of the transformer.

TRS1

Solution:

Complete the calculations in the following table.

1		2		3	4	5	6	7	8
Period Hrs	×	Pout kW	=	Energy Out PoutxTime kWh	IRON P_{oc} kW	Losses P_{sc} copper kW	Losstotal kW	Pin(tot) kW	Energy In PinxTime kWh
8		80 (100kVA) n (full load)		80x8 = 640	0.5	0.75	1.25	81.25	81.25x8 = 650
6		50x0.9 = 45 (50kVA) n (½ load)		45x6 = 270	0.5	0.1875	0.6875	45.6875	274.125
4		20 (25kVA) n (¼ load)		20x4 = 80	0.5	0.047	0.547	20.547	82.2
3		0 (energised) n (no load)		0	0.5	0	0.5	0.5	0.5x3 = 1.5
3		0 (de-energised)		0	0	0	0	0	0
		TOTAL		990kWh				TOTAL	1007.8kwh

Notes: Copper loss calculations (P_{SC}) for fractional load conditions must use

$P = n^2 x P_{SCFL}$'

The value of "n" used, is determined by the fraction of full load kVA supplied not the fraction of kW supplied.

All Day Efficiency % = $\frac{\text{Energy output for 24 Hrs x 100}}{\text{Energy Input for 24 Hrs}}$ = $\frac{\text{kWh out}}{\text{kWh in}}$

= $\frac{990 \times 100}{1007.8 \text{ kwh}} \times 100$

= 98.23%

Energy output kWh = col 2 × col 1

Total Losses = Total Poc + Psc - col 4 + col 5

Total input = Total output + total losses - col 3 + col 6

Pin Energy hours = Pin × hours = 7 × 1

TRS2

PARALLEL OPERATION OF SINGLE PHASE TRANSFORMERS

Industrial loads often vary from heavy load to light load over a 24 hour period.

It is uneconomical to have a large transformer working for a large proportion of the time on light load.

Several transformers can be connected in parallel to share the load and can be switched in as load increases.

Connection of Transformers in Parallel

Refer to FIG 1 which shows two single phase transformers connected in parallel.

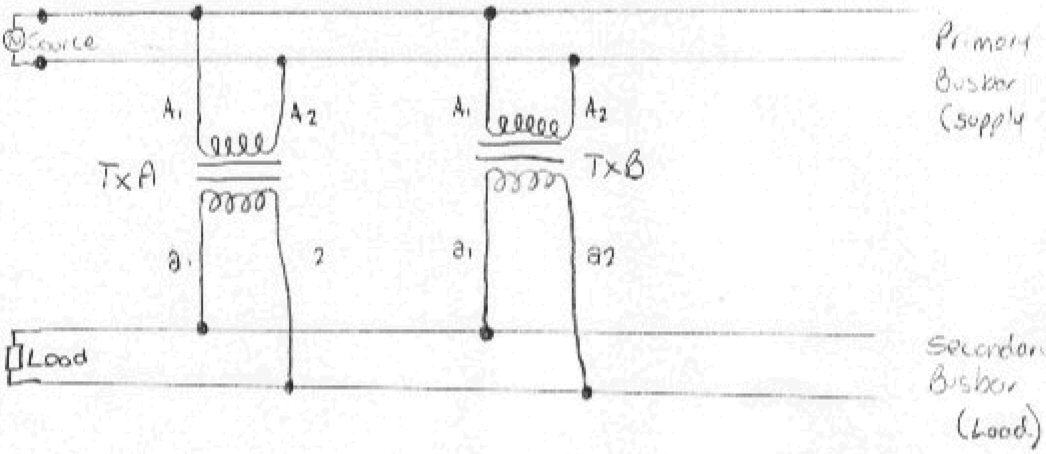


FIG 1

The primary windings are connected in parallel across the supply, and the secondary windings are connected in parallel to supply the load.

Conditions for Parallel Operation

Before transformers are connected in parallel, the following conditions must be satisfied.

a) the voltage ratio of the transformers must be the same,

b) the impedance triangles and % impedance of the transformers should be the same,

c) terminals of like polarity must be connected together.

TRS3

If the transformers have different voltage ratios, large short circuit currents or circulating currents will flow when the secondary windings are connected together.

Out of phase voltages due to incorrect polarity will also cause either large short circuit currents or circulating currents to flow when connections are made.

If the transformer winding impedances are different, then the transformers will not share the load in proportion to their kVA ratings.

Polarity Test for Parallel Transformers

Although the terminals of transformers are usually identified with polarity markings, correct polarity must be checked before paralleling the transformers.

Refer to FIG 2 which shows two transformers whose primary windings are paralleled across the supply, but whose secondaries are paralleled through a voltmeter.

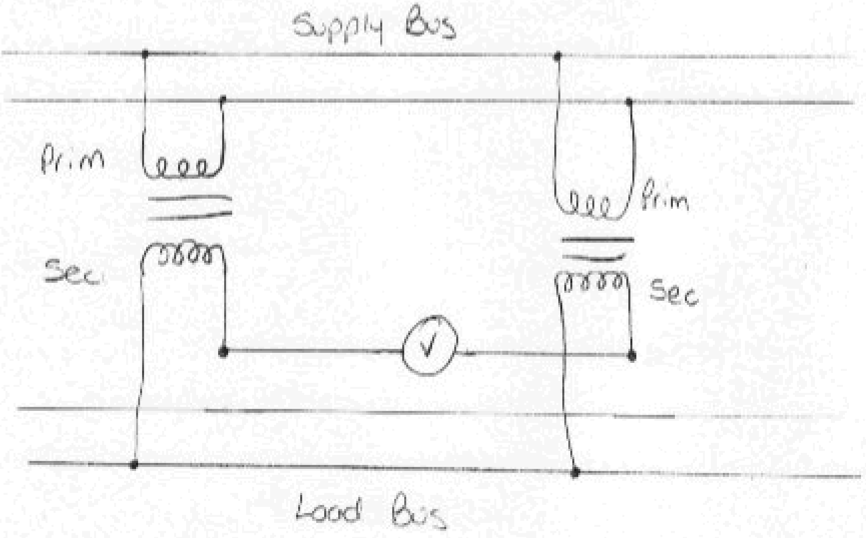


FIG 2

If the terminals to which the voltmeter are connected are of the same polarity, then the voltmeter will read zero, and the secondary windings can be safely paralleled.

If the terminals to which the voltmeter are connected are not of the same polarity, then the voltmeter will read two times the secondary voltage of the transformers, and the secondary windings must not be paralleled.

Note: Only a small difference in the two secondary voltages will cause circulating currents between the two windings and copper losses in the transformer even if no load is connected.

Load Sharing by Transformers in Parallel

The proportion of total load supplied by each transformer in parallel, depends on the winding impedance of each transformer.

This is similar to the current divider rule for current distribution, since power is proportional to current.

However, before applying the divider rule, the winding impedances must be referred to the same base kVA or MVA.

If two transformers "A" and "B" with impedances Z_A and Z_B are connected in parallel, then the total load VA is shared between the two transformers according to the following equations.

$$\text{Load on } TX_A = \frac{VA_{TOTAL} \times Z_B}{Z_A + Z_B}$$

$$\text{Load on } TX_B = \frac{VA_{TOTAL} \times Z_A}{Z_A + Z_B}$$

Notes: These calculations should be done using complex numbers.
The impedance values can be expressed either in Ω or %.

Example: A 500kW load with a power factor of 0.85 lagging, is supplied by two transformers in parallel.
TX_A is rated at 800kVA with $Z = 3 + j5\ \Omega$,
TX_B is rated at 400kVA with $Z = 2 + j4\ \Omega$.

Calculate the loading on each transformer.

Solution:

$$\text{Total load in kVA} = \frac{kW}{pf} = \frac{500}{0.85} = 588\text{kVA}$$

$$\theta = \cos^{-1} 0.85 = 31.8°$$

$$\text{Total Load } S_{TOTAL} = 588\underline{/+31.8°}\ \text{kVA (note positive angle)}$$

The transformer impedances must be corrected to the same base kVA, in this case choose a base of 800kVA.

TR55

Put in complex form

For TX_A Z_A = $3 + j5\,\Omega$ on a base of 800kVA (rating)

3 + shift R→P 5 = 5.83 ⇒ shift x↔y = θ = 59°

= $5.83/\underline{59^\circ}\Omega$

For TX_B Z_B = $2 + j4\,\Omega$ on a base of 400kVA (rating)

= $\frac{(2 + j4) \times 800}{400}$ (on a base of 800kVA)

= $4 + j8\,\Omega$ on a base of 800kVA

= $8.94/\underline{63.4^\circ}\Omega$

$Z_A + Z_B$ = $(3 + j5) + (4 + j8)$

= $7 + j13$

= $14.76/\underline{61.7^\circ}\Omega$

Load on TX_A = $\frac{kVA_{TOTAL} \times Z_B}{Z_A + Z_B}$

= $\frac{588/31.8^\circ \times 8.94/63.4^\circ}{14.76/\underline{61.7^\circ}}$

= $356.4/\underline{33.5^\circ}$kVA

= 297.2kW at 0.834 pf lagging

Load on TX_B = $\frac{kVA_{TOTAL} \times Z_A}{Z_A + Z_B}$

= $\frac{588/31.8^\circ \times 5.83/59^\circ}{14.76/\underline{61.7^\circ}}$

= $232.2/\underline{29.1^\circ}$kVA

= 202.9kW at 0.873 pf lagging

Notes: Total load adds to 500kW.
Although the rating of TX_A is twice the rating of TX_B, the transformers have not shared the load according to their ratings.

If these two transformers were supplying a larger load, TX_B would overload before TX_A.

TRS6

AUTOTRANSFORMERS

A conventional double wound transformer is constructed with two electrically separate windings wound on a common magnetic core.

However, an autotransformer consists of a single winding on a common magnetic core.

Refer to FIG 1 which shows a connection diagram of a step-down autotransformer.

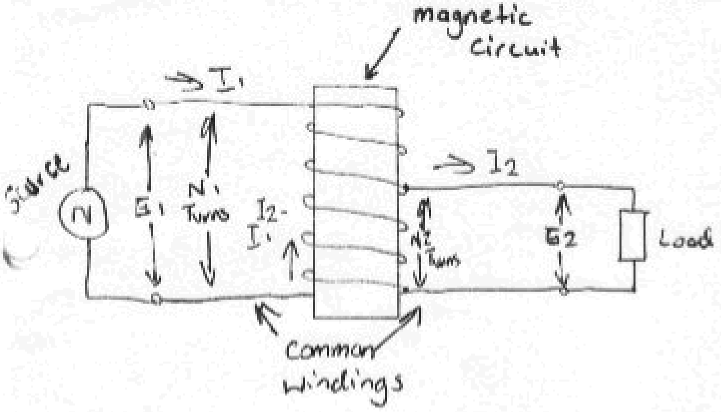


FIG 1

One pair of terminals is connected across the whole winding (N_1 turns) which is called the primary winding.

Another pair of terminals is connected to one end of the winding and to a tapping on the winding (N_2 turns) and this is called the secondary winding.

This part of the winding (N_2 turns) is called the "common" because it is common to both the primary and the secondary while the remainder ($N_1 - N_2$ turns) is only part of the primary winding.

Like a double wound transformer, the following equations also apply to the unloaded step-down autotransformer:

$$\boxed{\frac{E_1}{E_2} = \frac{N_1}{N_2} = k}$$

For the step-down autotransformer, k>1.

When a load is connected to the autotransformer secondary winding (N_2), and draws current I_2, the corresponding primary current is I_1.

TRS7

Using Kirchhoff's Current Law, the current in the "common" winding is $(I_2 - I_1)$ flowing up.

Since MMF in the magnetic core must balance, then:

$$I_1 x(N_1 - N_2) = (I_2 - I_1)xN_2$$

$$N_1 I_1 - N_2 I_1 = N_2 I_2 - N_2 I_1$$

which reduces to:

$$\boxed{\frac{I_1}{I_2} = \frac{N_2}{N_1} = \frac{1}{k}}$$

Note: This is the same equation as for a normal double wound transformer.

Some of the load current comes from the supply through the primary winding, and the remainder of the load current comes from the common winding through transformer action.

Comparison of Fully Wound and Autotransformers

Assume that the fully wound transformer and the autotransformer have the same voltage ratios V_1/V_2, supply the same load current I_2 (and therefore the same load VA) and are both ideal.

The Fully Wound Transformer has two separate windings, one with N_1 turns capable of carrying current I_1, and the other with N_2 turns capable of carrying current I_2.

The Autotransformer has a single winding with N_1 turns with a tapping at N_2 turns, the separate part of the winding ($N_1 - N_2$ turns) carrying current I_1 downward, and common part of the winding (N_2 turns) carrying only current $(I_2 - I_1)$ upward.

This comparison, shows that the autotransformer saves one winding (N_2 turns), and the common part of the winding can be wound using smaller cross section conductor.

This means that an autotransformer will have less windings than a double wound transformer for the same VA rating.

VA Rating of Windings for Step-down Transformers (k>1)

If the transformers are ideal and have no losses, then $V_1I_1 = V_2I_2$.

Fully Wound:

$$\text{Total VA rating of Transformer} = VA_{PRIMARY} + VA_{SECONDARY}$$

$$= V_1I_1 + V_2I_2$$

$$= 2V_1I_1$$

Autotransformer:

$$\text{Total VA rating of Transformer} = VA_{SEPARATE} + VA_{COMMON}$$

$$= (V_1 - V_2)I_1 + V_2(I_2 - I_1)$$

$$= V_1I_1 - V_2I_1 + V_2I_2 - V_2I_1$$

$$= V_1I_1 + V_2I_2 - 2V_2I_1$$

Since $V_1I_1 = V_2I_2$

then Total VA $= 2V_1I_1 - 2V_2I_1$

Comparing VA ratings:

$$\frac{\text{VA of Autotransformer}}{\text{VA of Double Wound}} = \frac{2V_1I_1 - 2V_2I_1}{2V_1I_1}$$

$$= \frac{2V_1I_1}{2V_1I_1} - \frac{2V_2I_1}{2V_1I_1}$$

$$= 1 - V_2/V_1$$

$$= 1 - 1/k \text{ (where } k>1)$$

This means that the autotransformer requires 1/k less windings than a double wound transformer to supply the same VA load with the same step-down voltage ratio.

Lower VA output

There is a saving in copper required for the autotransformer, and the weight of copper in the winding is proportional to the winding VA rating.

Copper Saving with Step-down Autotransformer = 1/k where k>1.

The Step-up Autotransformer

Refer to FIG 2 which shows a connection diagram of a step-up autotransformer.

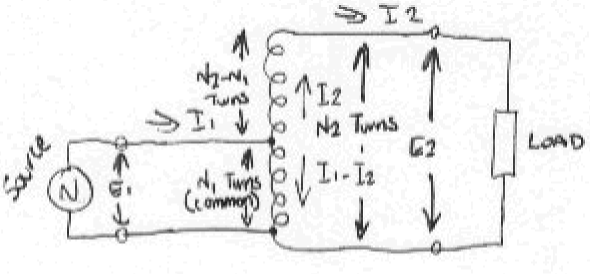


FIG 2

This Autotransformer has a single winding with N_2 turns with a tapping at N_1 turns, the separate part of the winding (N_2 - N_1 turns) carrying current I_2 upward, and common part of the winding (N_1 turns) carrying only current (I_1 - I_2) downward.

Again, this comparison, shows that the autotransformer saves one winding (N_1 turns), and the common part of the winding can be wound using smaller cross section conductor.

The step-up autotransformer will also have less windings than a step-up double wound transformer for the same VA rating.

Like a double wound transformer, the following equations also apply to the unloaded step-up autotransformer:

$$\boxed{\frac{E_1}{E_2} = \frac{N_1}{N_2} = k'}$$

k' = step up

For the step-up autotransformer, k'<1.

When a load is connected to the autotransformer secondary winding (N_2), and draws current I_2, the corresponding primary current is I_1.

TR60

Using Kirchhoff's Current Law, the current in the "common" winding is $(I_1 - I_2)$ flowing down.

Since MMF in the magnetic core must balance, then:

$$\begin{aligned}(N_2 - N_1)I_2 &= N_1(I_2 - I_1)\\ N_2I_2 - N_1I_2 &= N_1I_2 - N_1I_1\end{aligned}$$

which reduces to:

$$\boxed{\frac{I_1}{I_2} = \frac{N_2}{N_1} = \frac{1}{k'}}$$

Note: This is the same equation as for a normal double wound step-up transformer.

Some of the load current comes from the supply, and some from the common winding through transformer action.

VA Rating of Windings for Step-up Transformers (k'<1)

If the transformers are ideal and have no losses, then $V_1I_1 = V_2I_2$.

Fully Wound:

$$\begin{aligned}\text{Total VA rating of Transformer} &= VA_{PRIMARY} + VA_{SECONDARY}\\ &= V_1I_1 + V_2I_2\\ &= 2V_1I_1\end{aligned}$$

Autotransformer:

$$\begin{aligned}\text{Total VA rating of Transformer} &= VA_{SEPARATE} + VA_{COMMON}\\ &= (V_2 - V_1)I_2 + V_1(I_1 - I_2)\\ &= V_2I_2 - V_1I_2 + V_1I_1 - V_1I_2\\ &= V_2I_2 + V_1I_1 - 2V_1I_2\end{aligned}$$

Since $V_1I_1 = V_2I_2$

then Total VA $= 2V_1I_1 - 2V_1I_2$

Comparing VA ratings:

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$$\frac{\text{VA of Autotransformer}}{\text{VA of Double Wound}} = \frac{2V_1I_1 - 2V_1I_2}{2V_1I_1}$$

$$= \frac{2V_1I_1}{2V_1I_1} - \frac{2V_1I_2}{2V_1I_1}$$

$$= 1 - I_2/I_1$$

$$= 1 - k' \text{ (where } k'<1)$$

This means that the autotransformer requires k' less windings than a double wound transformer to supply the same VA load with the same step-down voltage ratio.

There is a saving in copper required for the autotransformer, and the weight of copper in the winding is proportional to the winding VA rating.

Copper Saving with Step-up Autotransformer = k' where k'<1.

Winding Copper Saving as a Function of k

Refer to FIG 3 which shows the relationship between transformer voltage ratio "k" and the % winding copper saving, if an autotransformer is used instead of a fully wound transformer.

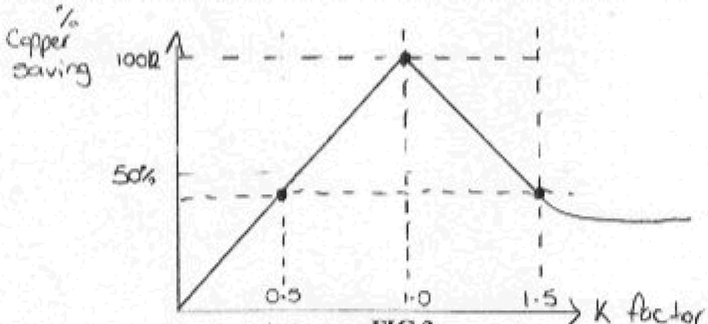


FIG 3

Advantage of the Autotransformer

Autotransformers are cheaper to build than double wound transformers of the same VA rating, since less windings are required. Cost, Weight, transport

Disadvantages of the Autotransformer

Unlike a double wound transformer, there is no electrical isolation between the primary and secondary windings.

If there is a large step-down in voltage, there is a risk of the high voltage supply appearing across the low voltage output if a short circuit occurs in the windings.

This means that the autotransformer cannot be used as an isolation transformer or where there is a <u>large</u> voltage ratio.

Applications of Autotransformers

1. Used to supply reduced voltage for starting squirrel cage induction motors (Autotransformer starting).
2. Used in HV interconnected power system substations to step down from 330kV to 132kV (three single phase units connected in star).
3. Not used for domestic or industrial supplies at distribution voltages, due to the hazard.
4. Voltage control of power and lighting circuits.

Connection of a Double Wound Transformer as an Autotransformer

A double wound transformer can be used as an autotransformer, by connecting the two windings in series to form a single autotransformer winding with a tapping.

Refer to FIG 4 which shows the series connections of the double wound transformer to produce an autotransformer.

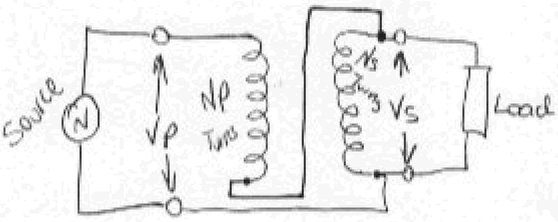


FIG 4

The VA rating of a double wound transformer connected as an autotransformer is <u>greater</u> than its rating as a double wound transformer.

Example: A 500/250V double wound single phase transformer is rated at 10kVA. The windings are connected in series, to form an autotransformer.

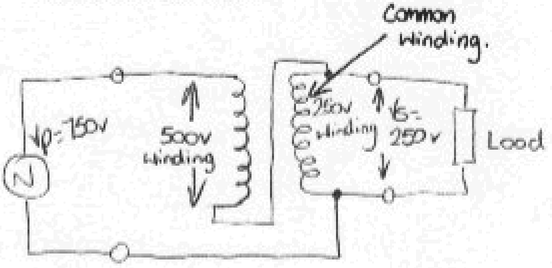
a) draw a diagram showing the connections when used as a 750/250V autotransformer,

b) calculate the rating of the autotransformer,

c) calculate the current flowing in each part of the autotransformer when it is supplying rated load.

Solution:

a)



b) As a double wound transformer, the rated current of each winding is

$$\text{Rated } I_{PRIMARY} = \frac{VA_{RATED}}{V_P} = \frac{10000\text{VA}}{500\text{V}} = \underline{20\text{A}}$$

$$\text{Rated } I_{SECONDARY} = \frac{VA_{RATED}}{V_S} = \frac{10000\text{VA}}{250\text{V}} = \underline{40\text{A}}$$

When connected as an autotransformer, the currents in each winding must be such that the mmfs in the core balance.

I_{PRIM} = 20A

I_{COMMON} = 40A

$I_{SECONDARY} = I_{PRIM} + I_{COMMON}$

$= 20 + 40 = 60A$

Output VA of Autotx = $V_S x I_S$ = 250x60

= 15kVA

Note: This rating is greater than the original 10kVA rating of the double wound transformer.

c) Current in common winding = 40A

Current in separate winding = 20A

THREE PHASE TRANSFORMER EQUIVALENT CIRCUIT

Three phase transformer calculations are carried out on a "single phase equivalent" circuit.

This means that the three phase transformer must be represented by a single phase equivalent and all quantities (voltage, current impedance, power etc) written as single phase equivalents.

Comparison of Single and Three Phase Transformers

Single Phase Transformer

$E_{PRIMARY}$ = Voltage across the primary winding

$E_{SECONDARY}$ = Voltage across the secondary winding

$I_{PRIMARY}$ = Current flowing in the primary winding

$I_{SECONDARY}$ = Current flowing in the secondary winding

Three Phase Transformer

$E_{PRIMARY\ LINE}$ = Voltage across the primary side lines

$E_{SECONDARY\ LINE}$ = Voltage across the secondary side lines

$E_{PRIMARY\ PHASE}$ = Voltage across a primary side winding

$E_{SECONDARY\ PHASE}$ = Voltage across a secondary side winding

$I_{PRIMARY\ LINE}$ = Current flowing in a primary line conductor

$I_{SECONDARY\ LINE}$ = Current flowing in a secondary line conductor

$I_{PRIMARY\ PHASE}$ = Current flowing in a primary winding

$I_{SECONDARY\ PHASE}$ = Current flowing in a secondary winding

Note: In Star connection $E_{LINE} = \sqrt{3}E_{PHASE}$

$I_{LINE} = I_{PHASE}$

In Delta connection $E_{LINE} = E_{PHASE}$

$I_{LINE} = \sqrt{3} x I_{PHASE}$

VA Watts and Vars

Single Phase Transformer

VA = $E_P x I_P$ or = $E_S x I_S$

Power = $E_P I_P \cos\theta$ or = $E_S I_S \cos\theta$

Vars = $E_P I_P \sin\theta$ or = $E_S I_S \sin\theta$

Three Phase Transformer

VA = $\sqrt{3} x E_{LINE} I_{LINE}$ (prim or sec)

Power = $\sqrt{3} x E_{LINE} I_{LINE} \cos\theta$ (prim or sec)

Vars = $\sqrt{3} x E_{LINE} I_{LINE} \sin\theta$ (prim or sec)

Turns Ratio of a Three Phase Transformer

The turns ratio of a transformer is the ratio of:

$$\frac{\text{Turns on a Primary Winding}}{\text{Turns on a Secondary Winding}}$$

and can be determined by the ratio of primary to secondary phase voltages.

Star-Star Transformer

Turns Ratio = $\frac{E_{PRIMARY\ PHASE}}{E_{SECONDARY\ PHASE}}$ or = $\frac{E_{PRIMARY\ LINE}}{E_{SECONDARY\ LINE}}$

Delta-Delta Transformer

Turns Ratio = $\frac{E_{PRIMARY\ LINE}}{E_{SECONDARY\ LINE}}$

Star-Delta Transformer

Turns Ratio = $\frac{E_{PRIMARY\ PHASE}}{E_{SECONDARY\ LINE}}$

Delta-Star Transformer

Turns Ratio = $\frac{E_{PRIMARY\ LINE}}{E_{SECONDARY\ PHASE}}$

Equivalent Circuit

The equivalent circuit of a three phase transformer is determined in a similar way to a single phase transformer except that the values are calculated **per phase**.

Short Circuit and No Load Tests

The short circuit and no load tests are carried out as a three phase test, but the test results are converted to single phase equivalents to allow equivalent resistance and reactance values to be calculated for each phase.

Example: A 10 MVA three phase, star-star connected transformer has a voltage ratio of 33kV/11kV.
A no load test is carried out on the transformer by applying 11kV to the secondary winding and recording the following results.

Line voltage = 11kV
Line Current = 15A
Power = 75kW

A short circuit test is carried out on the transformer by applying a test voltage to the primary winding with a short circuit applied to the secondary winding and recording the following results.

Line voltage = 1650V line-line
Line current = IRATED
Power = 90kW

Draw the equivalent circuit of the transformer with all components referred to the primary side.

Solution:

No load test results per phase:

$$E_{TEST} = 11kV/\sqrt{3} = 6.35kV$$

$$I_{TEST} = 15A$$

$$P_{TEST} = 75kW/3 = 25kW.$$

$$R_{0S} = E_0{}^2/P_0$$

$$= \frac{(6.35x10^3)^2}{25x10^3}$$

$$= 1613\Omega$$

$$I_{R0} = E_0/R_0$$

$$= \frac{6.35x10^3}{1613}$$

$$= 3.94A$$

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$$I_{X0} = \sqrt{(I_0^2 - I_{R0}^2)}$$

$$= \sqrt{(15^2 - 3.94^2)}$$

$$= 14.47\text{A}$$

$$X_{0S} = E_0/I_{X0}$$

$$= \frac{6.35\text{x}10^3}{14.47}$$

$$= 438.8\Omega$$

$$\text{Transformer turns ratio} = \frac{E_{P\ LINE}}{E_{S\ LINE}}$$

$$= \frac{33\text{kV}}{11\text{kV}}$$

$$= 3.$$

$$R_{0P} = R_{0S}\text{x}(3)^2$$

$$= 1613\text{x}9$$

$$= 14.5\text{k}\Omega$$

$$X_{0P} = X_{0S}\text{x}(3)^2$$

$$= 438.8\text{x}9$$

$$= 3.95\text{k}\Omega$$

Short circuit test results per phase:

$$E_{TEST} = 1650/\sqrt{3} = 953\text{V}$$

$$I_{TEST} = \frac{\text{Rated 3phase VA}}{\sqrt{3}\text{x}E_{LINERATED}}$$

$$= \frac{10\text{x}10^6}{\sqrt{3}\text{x}33\text{x}10^3}$$

$$= 175\text{A}$$

$$P_{TEST} = 90\text{kW}/3 = 30\text{kW}.$$

$$R_{EP} = \frac{P_{TEST}}{I_{TEST}^2}$$

TVR 73

$$= \frac{30\text{x}10^3}{175\text{x}175}$$

$$= 0.98\Omega$$

$$Z_{EP} = \frac{E_{TEST}}{I_{TEST}}$$

$$= \frac{953}{175}$$

$$= 5.4\Omega$$

$$X_{EP} = \sqrt{(Z_{EP}{}^2 - R_{EP}{}^2)}$$

$$= \sqrt{(5.4^2 - 0.98^2)}$$

$$= 5.3\Omega$$

TR64

POWER TRANSFORMERS $\qquad$ 3ϕ VECTOR GROUPINGS

CONNECTIONS FOR TRANSFORMERS.

With windings that can be connected in star, delta mesh, zigzag, with primary, secondary, tertiary, or auto connections; and transformers in single units or in banks of three, it is clear that the variety of connections is very great. No attempt will be made here to describe them completely: in many cases the characteristics, advantages, and drawbacks of a given type of connection can be estimated from the vector of the primary and secondary e.m.f. s.

A vector diagram can be constructed on the following general principles -

(a) The voltages of corresponding primary and secondary windings on the same limb (i.e., the input or applied primary voltage, and the developed secondary output voltage) are in phase opposition and the two induced e.m.f. s are in phase.

(b) The e.m.f. s induced in the three phases are equal, balanced, displaced mutually by one-third period in time, and have a definite sequence.

Nomenclature.

Transformer terminals . are brought out in rows, the h.v. on one side and the l.v. on the other, and are lettered from left to right facing the h.v. side. The h.v. terminals have capital letters (e.g. ABC); and l.v. terminals corresponding having small letters (e.g. abc). Tertiary windings, where provided, are lettered with capitals enclosed in circles. Neutral terminals precede line terminals. Each winding has two ends designated by the subscript numbers 1. 2; or if there are intermediary tappings, these are numbered in order of their separation from end 1. Thus an h.v. winding on phase A with four tapping would be numbered A_1, A_2, A_3... A_6, with A_1 and A_6 forming the phase terminals.

If the induced e.m.f. in an h.v. phase A_1 A_2 be in the direction A_1 to A_2 at a given instant, then the induced e.m.f. in the corresponding l.v. phase at the same instant will be from a_1 to a_2. The vector diagram in Fig. 1 represent induced e.m.f.'s (not applied voltages) for a number of methods of connections.

Polyphase transformers are allotted symbols giving the type of phase connection and the angle of advance turned through in passing from the vector representing the h.v. e.m.f. to that representing the l.v. e.m.f. at the corresponding terminal. The angle may be indicated by a clockface hour figure, the h.v. vector being 12 o'clock (zero) and the correspondin l.v. vector being represented by the hour hand. Thus "Yzd 11" represents a (h.v. star/l.v. zigzag/tertiary delta) - connected 3-phase transformer, with the l.v. (secondary) e.m.f. vector in a given phase-combination at "11 o'clock," i.e. + 30° in advance of the 12 o'clock position of the h.v e.m.f.

The groups into which all possible three-phase transformer connectio are classified are -

Group 1: Zero phase displacement (yy0, Dd0, Dz0).

Group 2: 180° phase displacement (Yy6, Dd6, Dz6).

Group 3: 30° lag phase displacement (Dy1, Yd1, Yz1).

Group 4: 30° lead phase displacement (Dy11, Yd11, Yz11).

The principal features of a few of the more common connections are noted below:-

Star-Star. (Yy0 or Yy6)

This is the most economical connection for small, high-voltage transformers as the number of turns per phase and the amount of insulation is a minimum. The possibility of utilizing both star points for a fourth wire may be useful.

Third-harmonic voltages are absent from the line voltage; unless there is a fourth wire no third-harmonic currents will flow. If the transformer is worked at normal flux density, however, the neutral potentials will oscillate, while the third-harmonic phase-voltages may be high for shell-type three-phase units. The connection is most satisfactory with the three-phase coretype: for other types the provision of a tertiary winding stabilizes the neutral conditions.

Delta-Delta (Dd0 or Dd6)

This is an economical connection for large, low-voltage transformers in which the insulation problem is not urgent, as it increases the number of turns per phase and reduces the necessary sectional area of conductors. Large unbalance of load can be met without difficulty, while the closed mesh serves to damp out third-harmonic voltages. It is possible to operate the transformer on 58 per cent of its normal rating in vee connection should one of the phases develop a fault. This, however, is not usually practicable with three-phase units. The absence of a star-point may be disadvantageous.

Star-Delta. (Dy or Yd)

The star-delta arrangement is very common for power-supply transformers. It has the advantage of a star-point for mixed loading, and a delta winding to carry third-harmonic currents which stabilize the star-point potential. If the h.v. winding is the star-connected side, there is some saving in cost of insulation (see Star-Star). A delta-connected h.v. winding is almost universal, however, when it is desired to work motors and lighting from a four-wire l.v. side.

Zigzag-Star. (Yz1 or Yz11)

The interconnection between phases effects a reduction of third-harmonic voltages and at the same time permits of unbalanced loadings; on account of the type of connection, however, the zigzag has to be confined to a fairly low-voltage winding. Since the phase voltages are composed on the zigzag side of two half-voltages with a phase difference of 60, 15 per cent more turns are required for a given total voltage per phase compared with a norma phase connection, which may necessitate an increase in the frame size over that normally used for the rating. The zigzag-star connection has been employed where delta connections were mechanically weak (on account of large numbers of turns and small copper sections) in high-voltage transformers; also for rectifiers.

General Remarks on Three-Phase Connections.

In three-phase working, which is becoming universal, a choice is possible between a three-phase unit and the bank of three single-phase units. A three-phase unit will cost about 15 per cent less than a bank, and will occupy considerably less space; this is reflected in power and substation building costs. There is no difference in reliability, but, as regards spare plant, it is cheaper to carry a single-phase than a three-phase unit if only one installation is concerned. Where there are several sets, this is less important. Single-phase banks are preferred in mines on account of the easier transport underground.

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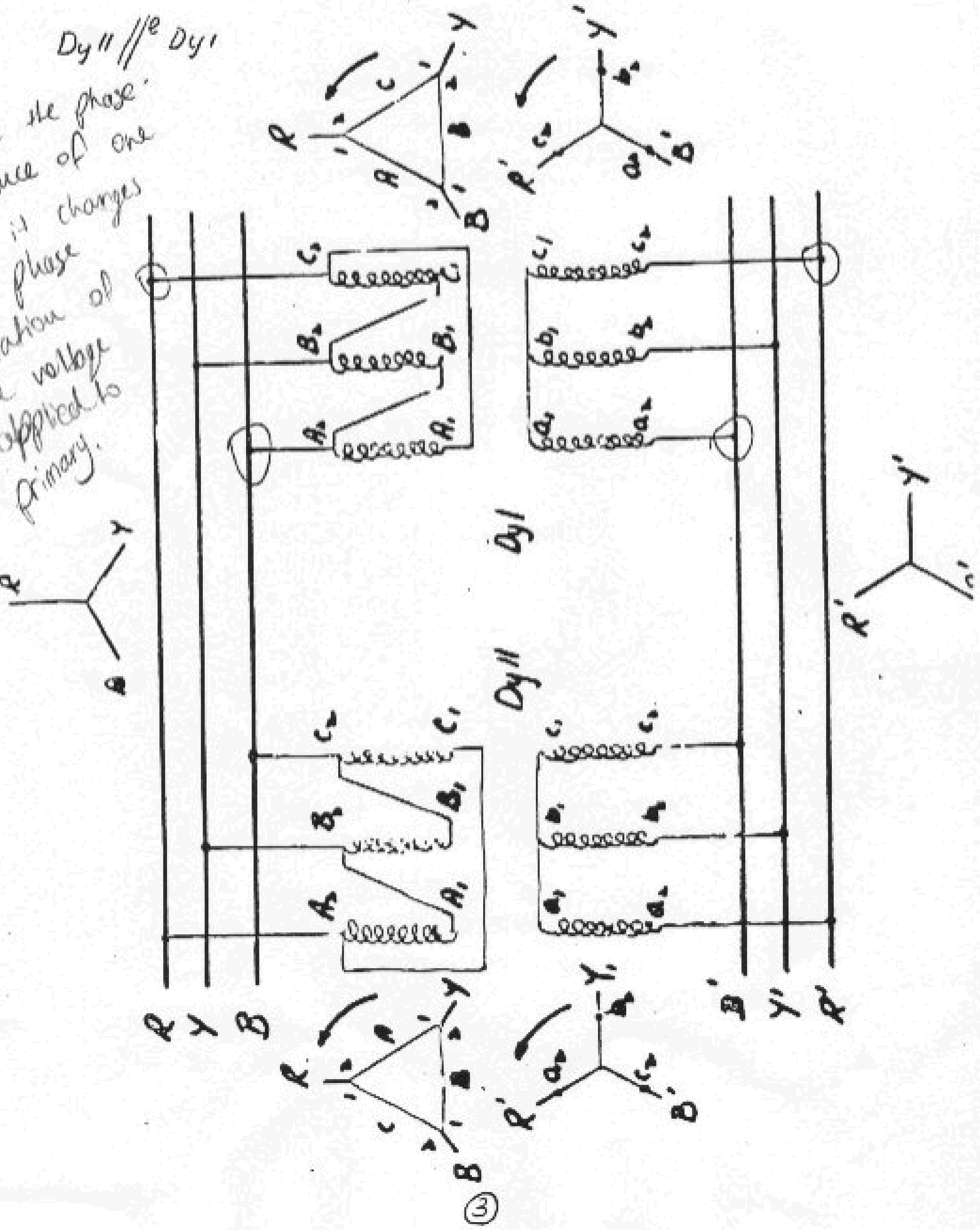
The choice between star and mesh connection merits separate consideration in each case. Star connection is cheaper, since mesh connection needs more turns and more insulation. The difference is small, however, at voltages below 11 kV. With very high voltages a saving of 10 per cent may be effected, mainly on account of the insulation. An advantage of the star-connected winding with earthed neutral is that the maximum voltage to the core (frame or earth) is limited to 58 percent of the line voltage, whereas with a delta-connected winding the earthing of one line (due to fault) increases the maximum voltage between windings and core to the full line voltage. Technically the mesh-connected primary is essential where the l.v. secondary is a star-connected four-wire supply to mixed three-phase and single-phase loads.

Dy11 //e Dy1

change the phase sequence of one tx. it changes the phase rotation of the voltage applied to primary.



Star Y

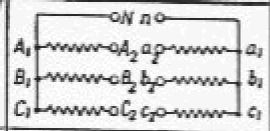
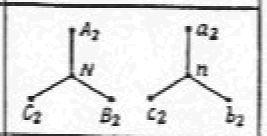
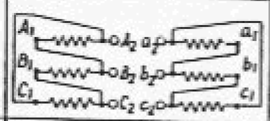
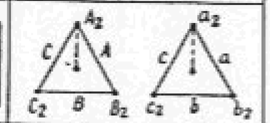
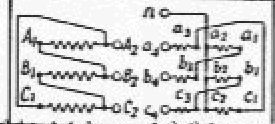
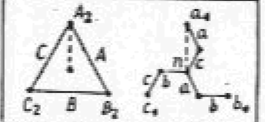
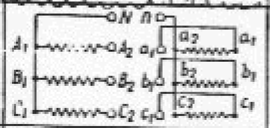
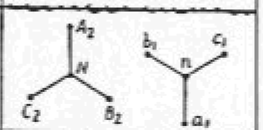
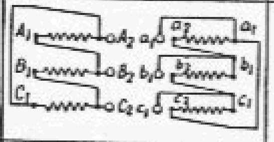
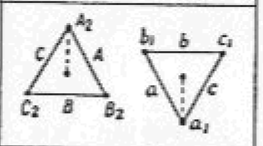
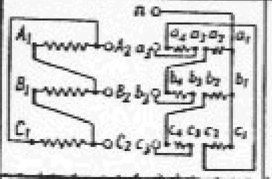
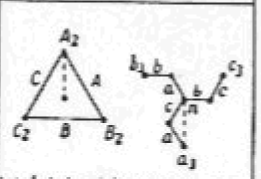
Delta D

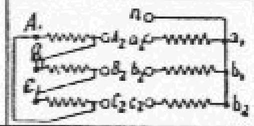
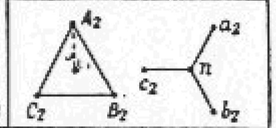
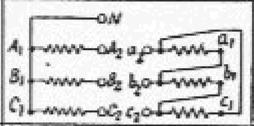
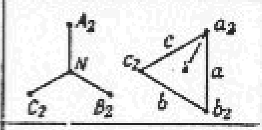
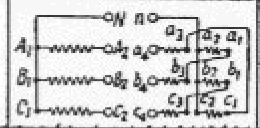
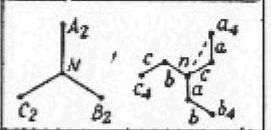
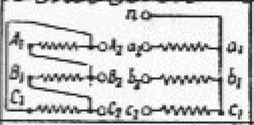
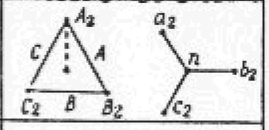
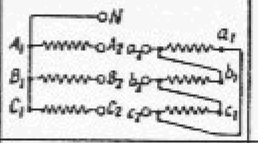
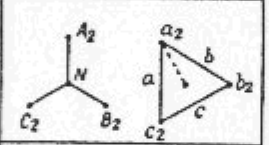
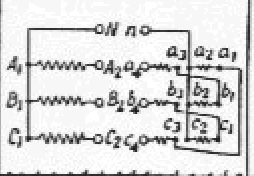
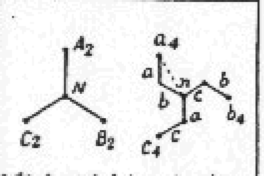
Zigzag Z

Vector grouping.

Ⓕ

shift.

GROUP, NO SYMBOL PHASE ∠	WINDINGS AND TERMINALS	E M F VECTOR DIAGRAMS
1_1 Yy0 0°		
1_2 Dd0 0°		
1_3 Dz0 0°		
2_1 Yy6 180°		
2_2 Dd6 180°		
2_3 Dz6 180°		

GROUP, NO SYMBOL PHASE ∠	WINDINGS AND TERMINALS	E. M. F. VECTOR DIAGRAMS
3_1 Dy1 -30°		
3_2 Yd1 -30°		
3_3 Yz1 -30°		
4_1 Dy11 +30°		
4_2 Yd11 +30°		
4_3 Yz11 +30°		

TR67

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THREE PHASE TRANSFORMER VOLTAGE REGULATION, LOSSES AND EFFICIENCY

The voltage regulation, losses and efficiency of a three phase transformer are calculated in a similar way to a single phase transformer, as the three phase transformer is represented as a single phase equivalent circuit.

Refer to separate notes on single phase transformer voltage regulation and losses and efficiency.

Example: A 1 000kVA, 6 600/415V, 3 phase, delta/star transformer has R% of 1.5, and X% of 4. Maximum efficiency occurs at 50% full load.

Calculate:

a) the iron loss

b) the full load efficiency at 0.8pf lagging

c) the maximum efficiency at 0.8pf lagging

d) the approximate percentage regulation on full load, unity power factor.

Solution:

Refer to FIG 1 which shows the circuit diagram of the transformer.

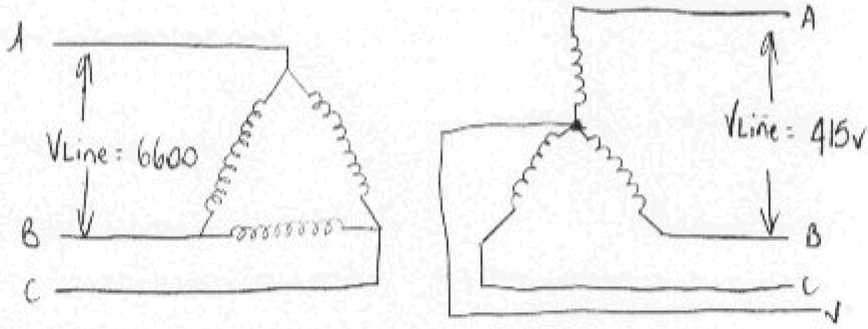

FIG 1

Full load primary line current is:

$$I_{\text{LINE PRIM FL}} = \frac{\text{3 Phase VA}}{\sqrt{3} \text{x} V_{\text{LINE}}}$$

TR75

$$= \frac{1000x10^3}{\sqrt{3}x6600}$$

$$= 87.5A$$

Full load primary phase current is:

$$I_{PHASE\ PRIM\ FL} = \frac{I_{LINE}}{\sqrt{3}}$$

$$= \frac{87.5}{\sqrt{3}}$$

$$= 50.5A$$

Equivalent resistance per primary phase is:

$$R_{EP} = \frac{R\%xV_{PHASE}}{I_{PHASE\ FL}x100}$$

$$= \frac{1.5x6600}{50.5x100}$$

$$= 1.96\Omega$$

a) Copper loss at ½ full load $= (I_{PHASE\ FL}/2)^2 \ x\ R_{EP}$ per phase

$= (50.5/2)^2 \ x\ 1.96$

= 1.25kW / phase

= 3.75kW for three phases.

Iron loss = Copper loss at maximum efficiency

Since maximum efficiency occurs at ½ full load, then

Iron loss = 3.75kW.

b) Full load copper loss $= I_{PHASE\ FL}^2 xR_{EP}$ per phase

$= 50.5^2 x1.96$

= 5kW / phase

= 15kW for three phases

Full load iron loss = 3.75kW for three phases

Efficiency at Full load 0.8pf lagging is:

$$\text{Full Load Efficiency \%} = \frac{\text{Full Load VA x Power Factor x 100}}{(\text{Full Load VA x Power Factor}) + P_{OC} + P_{SC}}$$

$$= \frac{1000\text{x}10^3\text{x}0.8\text{x}100}{(1000\text{x}10^3\text{x}0.8) + (3.75\text{x}10^3) + (15\text{x}10^3)}$$

$$= 97.71\%$$

c) Maximum efficiency occurs at 50% full load (n = 0.5)

Maximum efficiency at 0.8 pf lagging is:

$$\text{Maximum Eff \%} = \frac{\text{nxFull Load VA x Power Factor x 100}}{\text{nx(Full Load VA x Power Factor)} + P_{OC} + n^2 P_{SC}}$$

$$= \frac{0.5\text{x}1000\text{x}10^3\text{x}0.8\text{x}100}{0.5(1000\text{x}10^3\text{x}0.8) + 3.75\text{x}10^3 + (0.5^2\text{x}15\text{x}10^3)}$$

$$= 98.16\%$$

d) Percentage Regulation at Full load Unity power factor is:

$$\text{\% Regulation} = R\%\cos\theta + X\%\sin\theta$$

$$= 1.5\text{x}1 + 4\text{x}0$$

$$= 1.5\%$$

This means that if rated voltage is applied to the transformer primary windings, then the voltages measured across the secondary lines or phases will be 1.5% lower than the rated values for this loading.

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UNBALANCED LOADS ON THREE PHASE TRANSFORMERS

Core Fluxes in a loaded transformer

Primary and secondary magnetomotive forces (mmfs) in the magnetic core of a transformer, must always balance.

This means, that for a particular load current flowing in the secondary windings producing an mmf, there must be a current flowing in the primary windings that will produce exactly the same mmf to balance.

The transformer ratio equations $N_P/N_S = V_P/V_S = I_S/I_P$ suggest that the ratio of currents in a transformer is the reciprocal of the turns ratio.

This is the case for balanced loads, so that we can calculate the primary and secondary currents in corresponding phase windings by using these equations.

However, when unbalanced loads are connected to three phase transformers, it is possible that the ratio of currents I_S/I_P is not equal to the ratio of N_P/N_S in corresponding windings.

The primary and secondary mmfs must balance, for the transformer to operate correctly. If the currents in corresponding windings do not produce balancing mmfs, then the additional mmf required, must be produced by another winding, and must have the correct phase relationship.

Below are some examples of unbalanced loads, and the resulting distribution of currents through the transformer. All transformers are assumed to have line voltage ratios of 1:1, are supplied from a star connected generator and the single phase loads are 100A at unity power factor.

CASE 1

Star-star transformer with single phase load across 2 lines

Refer to FIG 1.
With this method of single phase loading, there are equal currents in the loaded phases and zero current in the unloaded phase.

The primary load currents have a free path through the two primary windings, corresponding to the loaded secondary phases and the two line conductors back to the generator.

There is therefore no choking effect, and the voltage drops in the transformer windings, are those due only to the normal impedance of the transformer.

The transformer neutral points are relatively stable, and the voltage of the open phase is practically the same as at no load. The secondary neutral point can be earthed without affecting the conditions.

The above remarks apply equally to all types of transformers whether they are of **core** type or **shell** type construction.

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CASE 2
Star-star transformer with single phase load from one line to neutral

Refer to FIG 2.
With this method of single phase loading, the primary current corresponding to the current in the loaded secondary, must find a return path through the other two primary phases. As load currents are not flowing in the secondary windings of these two phases, the load currents in the primaries act as magnetising currents to the two phases. This results in the voltages of the two unloaded phases increasing considerably while the voltage of the loaded phase decreases.

The neutral point, therefore, is considerably deflected.

The current distribution shown is only approximate, as this will vary with each individual transformer design.

The above remarks apply strictly to three phase shell type transformers and to three phase banks of single phase transformers.

Three phase core type transformers can, on account of their interlinked magnetic circuits, supply considerable unbalanced loads without very severe deflection of the neutral point.

CASE 3
Star-star transformer with generator neutral joined and single phase load from one line to neutral

Refer to FIG 3.
In this case, the connection between the generator and transformer neutral points, provides the return path for the primary load current, and this effectively short circuits the other two phases.

There is therefore no choking effect, and the voltage drops in the transformer windings, are those on the one phase only, due to the normal impedance of the transformer.

The transformer neutral points are relatively stable, and the voltages of the above phases are practically the same as at no load.

The secondary neutral point may be earthed without affecting the conditions.

The above remarks apply equally to all types of transformers.

CASE 4
Delta-delta transformer with single phase load across 2 lines

Refer to FIG 4.
With this connection, the loaded phase carries 2/3 of the total load current, while the remainder flows through the other two phases, which are in series with each other, and in parallel with the loaded phase.

On the primary side, all three windings carry load currents in the same proportion as the secondary windings, and two of the line conductors carry the current to and from the generator.

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There is no abnormal choking effect, and the voltage drops are due to the normal impedance of the transformer only.

The above remarks apply equally to all types of transformers.

CASE 5
Star-delta transformer with single phase load across 2 lines

Refer to FIG 5.
On the secondary delta side, the distribution of current in the transformer windings is 2/3 in the loaded phase and 1/3 in the other two phases.

On the primary side, the corresponding load currents are split up in the same proportions as on the secondary.

The primary currents are equal to the secondary currents of the different phases multiplied by √3, and multiplied by the ratio of transformation according to whether the transformer is step up or step down.

The primary neutral point is stable.

The above remarks apply equally to all types of transformers.

CASE 6
Delta-star transformer with single phase load across 2 lines

Refer to FIG 6
With this method of single phase loading, there are equal currents in the loaded phases and zero current in the unloaded phase.

Currents in the corresponding primary windings are 1/√3 or 58% of the secondary ratio currents.

There are currents flowing in all three lines back to the generator, with one line carrying twice the current in the other two.

There is no choking effect, and the voltage drops in the windings are due only to the normal impedance of the transformer.

The transformer secondary neutral point is relatively stable and may be earthed.

The voltage of the open phase is practically the same as at no load.

The above remarks apply equally to all types of transformers.

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CASE 7
Delta-star transformer with single phase load from one line to neutral

Refer to FIG 7.
With this connection, the load current flows only in the loaded phase and the neutral on the secondary.

On the primary side, 1/√3 times ratio current flows in the loaded phase and returns to the generator through two lines.

There is no choking effect, and the voltage drops in the windings are due only to the normal impedance of the transformer.

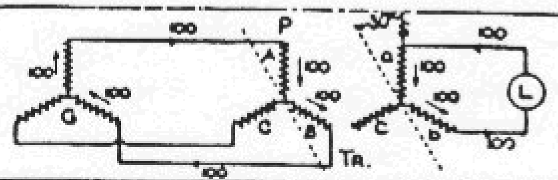
The secondary neutral point is stable and may be earthed without affecting the conditions.

The voltages of the open phase are practically the same as at no load.

The above remarks apply equally to all types of transformers.

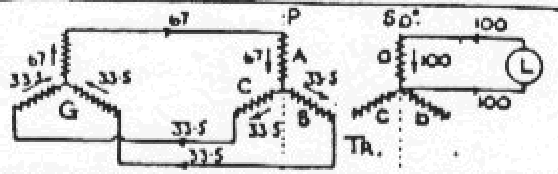
TR31

1.



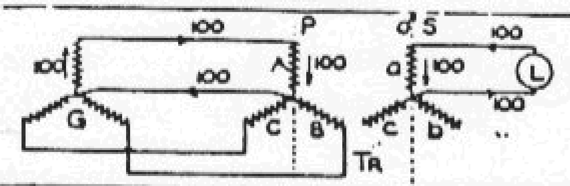
STAR/STAR
3-3 PHASE

2.



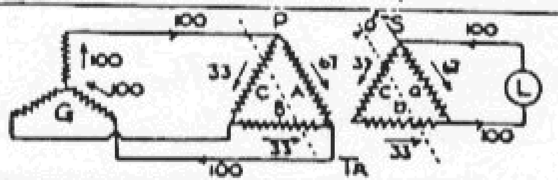
STAR/STAR
3-3 PHASE

3.



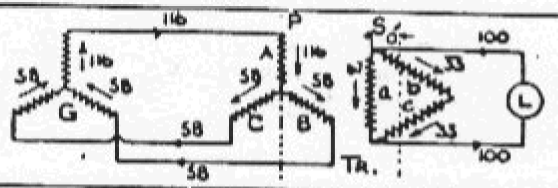
STAR/STAR
3-3 PHASE

4.



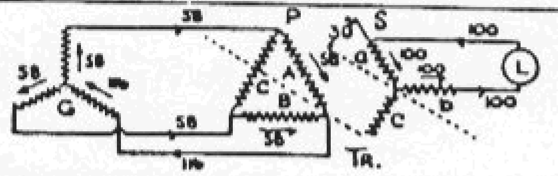
DELTA/DELTA
3-3 PHASE

5.



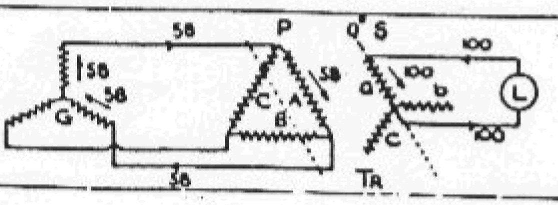
STAR/DELTA
3-3 PHASE

6.



DELTA/STAR
3-3 PHASE

7.



DELTA/STAR
3-3 PHASE

P13

TR82

PARALLEL OPERATION OF THREE PHASE TRANSFORMERS

If three phase transformers are to operate effectively in parallel, and share the load according to their ratings, then they must be identical in the following parameters.

a) Voltage Ratio
b) Vector Grouping
c) Percentage Impedance.

If any of these requirements are not met, then currents will flow between the secondary windings of the two transformers even when no load is connected, and the transformers will not share the total load.

Note: During testing, sometimes the voltage ratio of one transformer is deliberately changed (by changing its tap position) so that current will flow between the two transformers, thus allowing the transformers to be loaded without having the provide a load impedance.

Refer to separate notes on single phase transformer parallel operation.

Load Sharing by Parallel Three Phase Transformers.

Equations similar to those for single phase transformers are used.

If two transformers "A" and "B" with impedances Z_A and Z_B are connected in parallel, then the total load VA is shared between the two transformers according to the following equations.

$$\text{Load on } TX_A = \frac{VA_{TOTAL} \times Z_B}{Z_A + Z_B}$$

$$\text{Load on } TX_B = \frac{VA_{TOTAL} \times Z_A}{Z_A + Z_B}$$

Note: These calculations should be done using complex numbers.
The impedance values can be expressed either in Ω or % and referred to the same "base" VA.

Example: Two 3 phase transformers are connected in parallel on both high and low voltage sides to supply a load.
Transformer A rated at 10MVA has a % impedance of (2 + j5) % and the transformer B rated at 20MVA has a % impedance of (3 + j7.5) %.
Both impedances are expressed in terms of the rated quantities of each transformer.
Calculate:

a) the % impedance of the 20MVA transformer to a base of 10MVA,
b) MVA supplied by each transformer if the load is 15MVA at unity pf
c) whether it is possible for this transformer combination to supply 30MVA.

TVR83

Solution:

a) $Z_A = 2 + j5\%$

$= 5.38\underline{/68.2^\circ}\%$ on 10MVA base.

$Z_B = 3 + j7.5\%$ on 20MVA base

$= \frac{(3 + j7.5)}{2}$ on 10MVA base

$= 1.5 + j3.75\%$

$= 4.04\underline{/68.2^\circ}\%$ on 10MVA base.

b) Total Load $= 15\underline{/0^\circ}$ MVA

Load on $TX_A = \frac{VA_{TOTAL} \times Z_B}{Z_A + Z_B}$

$= \frac{15/0^\circ \times 4.04/68.2^\circ}{(2 + j5) + (1.5 + j3.75)}$

$= \frac{60.6/68.2^\circ}{3.5 + j8.75}$

$= \frac{60.6/68.2^\circ}{9.42\underline{/68.2^\circ}}$

$= 6.43\underline{/0^\circ}$ MVA

Load on $TX_B = \frac{VA_{TOTAL} \times Z_A}{Z_A + Z_B}$

$= \frac{15/0^\circ \times 5.38/68.2^\circ}{9.42\underline{/68.2^\circ}}$

$= 8.57\underline{/0^\circ}$ MVA

c) These transformers do not share load according to their ratings, so that they could not supply a total load of 30MVA without Transformer A overloading.

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HARMONICS IN TRANSFORMERS

The use of high flux densities in transformer cores is imposed by requirements of an economical design and the reduction of dead weight.

This often results in saturation of the magnetic circuit and operation in the non-linear part of the B-H curve. Figure 1 below, shows how magnetising current varies with time, for a sinusoidal flux waveform corresponding to a sinusoidal emf.

The current waveform is not sinusoidal and contains odd harmonics. The major harmonics present are third and fifth, which increase in amplitude as the flux density level increases.

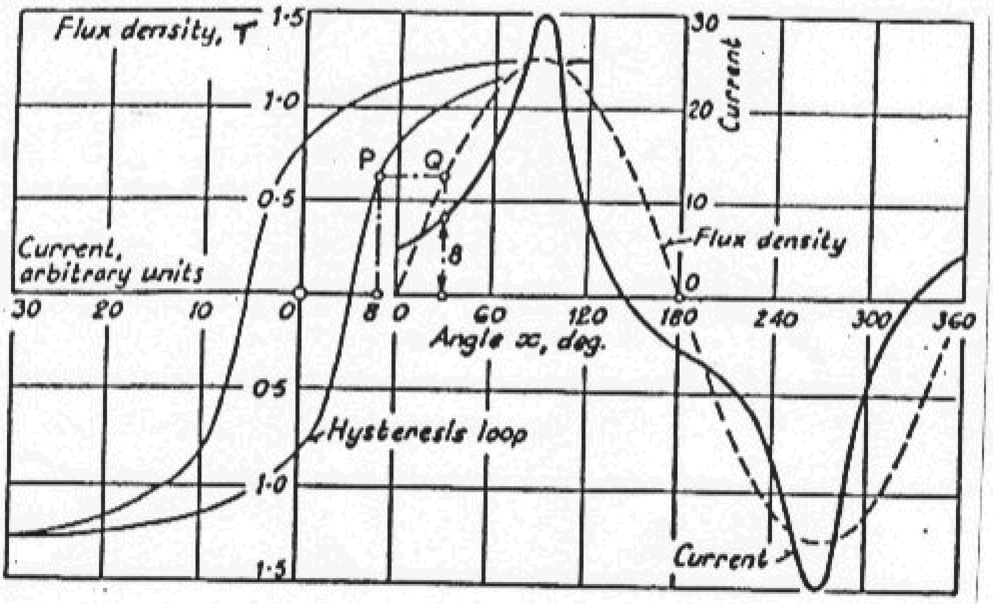

FIG 1

Thus a sinusoidal flux wave (required by a sinusoidal applied voltage) demands a magnetising current with a harmonic content.

But a supply of strictly sinusoidal voltage cannot supply a harmonic current.

If a sinusoidal magnetising current is provided, however, the flux wave will fail to reach its sinusoidal peak value and will become flat topped.

The emf induced by it will then become peaky with third and other harmonics. Figure 2 shows the relative shapes of flux, emf and magnetising current waveforms for conditions of sinusoidal emf and sinusoidal current.

Note that if emf is sinusoidal, then flux is also sinusoidal, whereas, if emf is distorted, then flux is also distorted.

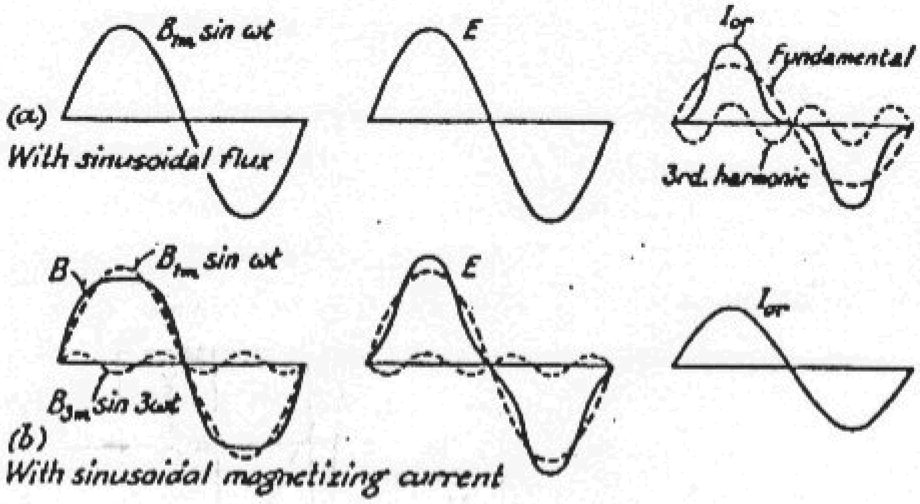

FIG 2

SINGLE PHASE TRANSFORMERS

Assuming a sinusoidal supply voltage, the flux will also be sinusoidal.

This means that magnetising current must be distorted and contain harmonics.

Note that this distortion can be seen in the current on no load, but may not be evident when the transformer is on load since it will be swamped by the load component of current which is much larger and sinusoidal.

The harmonic currents set up I^2R losses and also increase core losses, which are proportional to frequency.

PHASE RELATIONSHIPS OF HARMONICS IN THREE PHASE CIRCUITS

In a system of balanced three phase voltages, the fundamentals, and the fifth, seventh, eleventh and thirteenth harmonics all produce voltages displaced by 120°, and the third and ninth harmonics produce voltages that are in phase with each other in each phase.

THREE PHASE BANKS OF SINGLE PHASE TRANSFORMERS

The effects here will depend on whether the phases are magnetically separate, or whether they are magnetically (as well as electrically) linked.

Where three phase banks of single phase transformers are used, the magnetic circuits are obviously separate, and each core must itself produce the flux demanded by the conditions of the electrical connections, which then determine the flow of harmonic currents.

Dd Connection

The delta connection provides a path for the third harmonic currents to flow, driven by the harmonic emfs which are all in phase.

Each phase harmonic emf is absorbed by its own IZ drop, and therefore no third harmonic components of voltage will be seen at the line terminals.

The impedance to harmonic currents is usually small and so very small emfs are sufficient to circulate considerable harmonic currents.

Yd and Dy Connection without neutral

So long as there is no neutral connection, either of these connections will operate in the same way as the Dd connection.

Third harmonic currents will circulate in the delta, but will not flow in the star connection, as they are in phase and require the fourth wire to flow through.

Yy Connection without neutral

Current is forced to be sinusoidal since there is no path for third harmonic components to flow.

I N

This will produce distorted flux which will in turn induce distorted emfs into the secondary windings.

$V + \Phi$ distorted N

Yy Connection with neutral

The neutral connection carries the third harmonic components from all three phases which are all in phase and therefore the current in the neutral is three times the harmonic current of one transformer.

Tertiary Delta winding

If Yy transformers are provided with a delta connected tertiary winding, harmonic currents can circulate in the delta connected winding and reduce distortion of flux and induced emfs.

Sometimes the delta winding is provided exclusively for this purpose or it may be used to supply another load.

If either or both of the primary and secondary windings have neutral connections, these will compete with the tertiary winding for some of the harmonic current, the division of which depends on the relative impedances of the alternate paths.

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THREE PHASE TRANSFORMERS

The conclusions determined above, for three single phase transformers can be applied directly to the **shell** type three phase transformer, in which the magnetic circuits of the separate phases are complete in themselves and do not interact.

However, with the three limbed core, the phases are magnetically interlinked and any third harmonic fluxes that exist, are directed either all up or all down in the limbs together at any instant.

The return paths of these fluxes must therefore lie outside the cores through the air or oil and the walls of the tank.

These paths are of high reluctance, so that there is a strong tendency to retain a sinusoidal flux and emf.

Occasionally the third harmonic fluxes have been found to cause losses in the tank walls.

Five limbed transformers and end limbs provide a return path for third harmonic fluxes.

EFFECTS OF TRANSFORMER HARMONICS

The effects of harmonic currents are:

a) additional I^2R loss due to circulating currents
b) increased core loss
c) interference magnetically with communication circuits

and may cause:

a) increased dielectric stresses
b) electrostatic interference with communication circuits
c) resonance between the inductance of the transformer winding and the capacitance of a feeder to which it is connected.

HARMONIC GENERATORS

Harmonic currents and voltages are generated by a variety of types of equipment. This includes equipment in which the impedance varies during each half cycle of the applied emf and by equipment which generates a non-sinusoidal back emf.

Transformer magnetic circuits have non-linear B-H curves and hysteresis effects.

The variable permeability of the core, causes a change in inductance and hence inductive reactance of the winding as it passes through a cycle of magnetisation.

Other equipment that may produce harmonic effects includes, rotating electrical machines, gaseous discharge lamps (including fluorescent lamps), arc furnaces, rectifiers and loads controlled by phase angle firing of thyristors.

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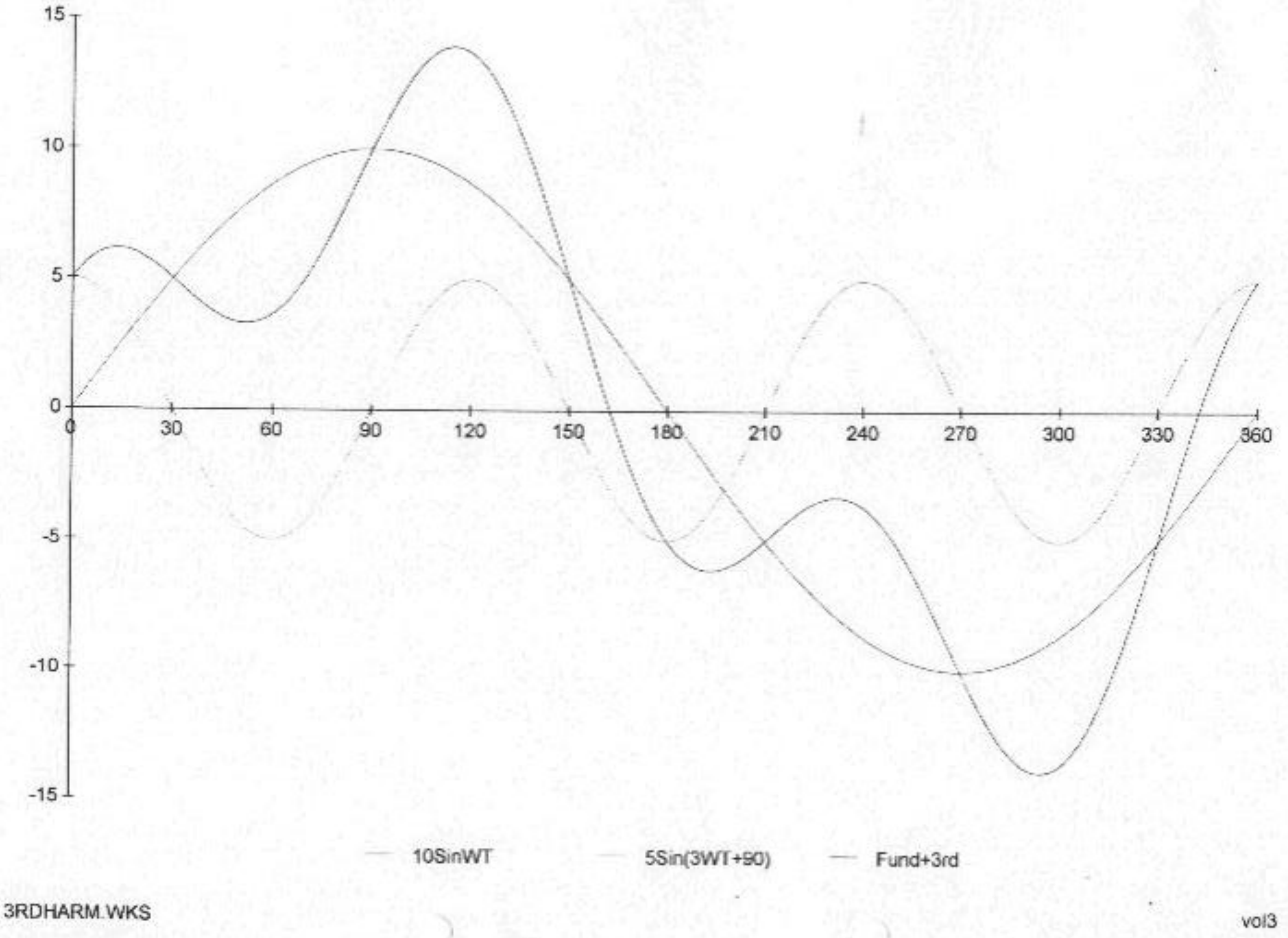
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5Sin(3WT+90)
Fund+3rd
3RDHARM.WKS
vol3