Dynamics Review Problems and Notes
Fundamentals of Engineering Exam

1. A sprinter competing in a 100 m race accelerates uniformly for the first 35 m in 5.4 sec. He then runs at a constant speed for the remainder of the race. He crosses the finish line in a time of: (circle one)

(a) 9.86 sec

(b) 10.05 sec

(c) 10.23 sec

(d) 10.41 sec

(e) 10.72 sec

Applicable Theory and Hints	Solution
Three Basic Kinematic Defining Equations: (1) $v = \frac{ds}{dt}$ (2) $a = \frac{dv}{dt}$ (3) $a ds = v dv$ **Integrate the defining eqns for a = constant :** (1) $v = v_0 + at$ (2) $s = s_0 + v_0 t + \frac{1}{2}at^2$ (3) $v^2 = v_0^2 + 2a(s - s_0)$ • *Remember: These equations are for cases where* ***acceleration is constant*** *only! If the acceleration is not constant, you must use the defining equations and integrate.*	This is a straight line motion problem where acceleration is constant. (2) $35 = 0 + 0 + \frac{1}{2}a(5.4)^2$ $\boxed{a = 2.40 \text{ m/s}^2}$ $V_{35} = 0 + 2.4(5.4)$ $= 12.96 \text{ m/s}$ Time to finish remaining 65 m: $t = \frac{65\text{m}}{12.96 \text{ m/s}}$ $t = 5.01$ sec Total Time: $5.4 + 5.01$ $= 10.41$ sec

2. A particle moves along a straight line with an acceleration of a = 2s, where s is in meters and a is in m/s^2. If the particle has a velocity of +2 m/s as it passes through the origin (s = 0), its velocity at s = 4 m will be: (circle one)

(a) 18 m/s

(b) 4.0 m/s

(c) 3.5 m/s

(d) 4.5 m/s

(e) 6 m/s

Solution: This is a problem where acceleration is not constant. You must use one of the defining equations above and integrate.

Use one of the defining equations: $a\,ds = v\,dv$

$$\int_0^s 2s\,ds = \int_{+2}^{v} v\,dv$$

$$s^2 = \frac{1}{2}v^2\Big|_2^v = \frac{1}{2}(v^2 - 4)$$

$$\boxed{v = \sqrt{4 + 2s^2}}$$

at $s = 4$ m:

$$v = \sqrt{4 + 32}$$

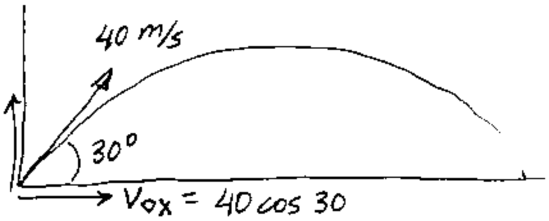
$$v = \pm 6 \text{ m/s}$$

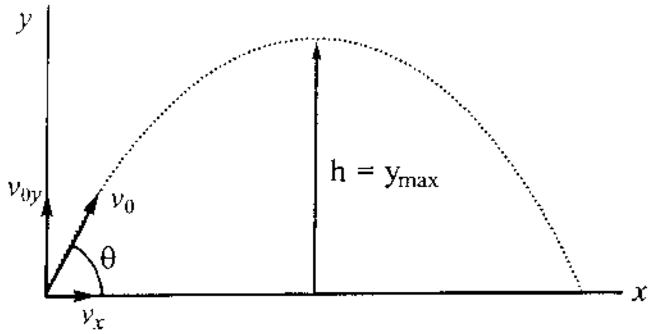
$$\boxed{v = +6 \text{ m/s}}$$

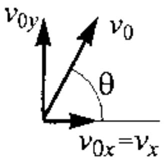
3. A projectile fired at 30° from the horizontal with an initial velocity of 40 m/s will reach a maximum height h above the horizontal of: (circle one)

(a) 81.5 m
(b) 20.4 m
(c) 6.2 m
(d) 24.8 m
(e) 141 m

$V_{0y} = 20$ m/s
$= 40 \sin 30$



Idealized Projectile Theory and Equations	**Solution**
	At max height, $V_y = 0$ ... $v_y^2 = v_{0y}^2 - 2g(\Delta y)$ $0 = 20^2 - 2(9.81)\,h$ $\boxed{h = \frac{400}{2(9.81)} = 20.4 \text{ m}}$

Coordinate Direction:	x	y
Acceleration:	$a_x = 0$	$a_y = -g$ *where,* $g = 9.81\ m/s^2$ in S.I. units *or,* $g = 32.2\ fps^2$ in U.S. units
Velocity:	$v_x = v_{ox} =$ *constant*	(2) $v_y = v_{0y} - gt$
Position:	(1) $x = x_0 + v_x t$	(3) $y = y_0 + v_{0y}t - ½ gt^2$
Additional equation:		(4) $v_y^2 = v_{0y}^2 - 2g(y - y_0)$
Calculating v_{0x} and v_{0y}, given v_0 and θ_0.		(5) $v_x = v_0 \cos \theta$ (6) $v_{0y} = v_0 \sin \theta$

4. Two automobiles shown below travel along a roadway. The relative acceleration ($\mathbf{a}_{B/A}$) of auto B with respect to A is: (circle one)

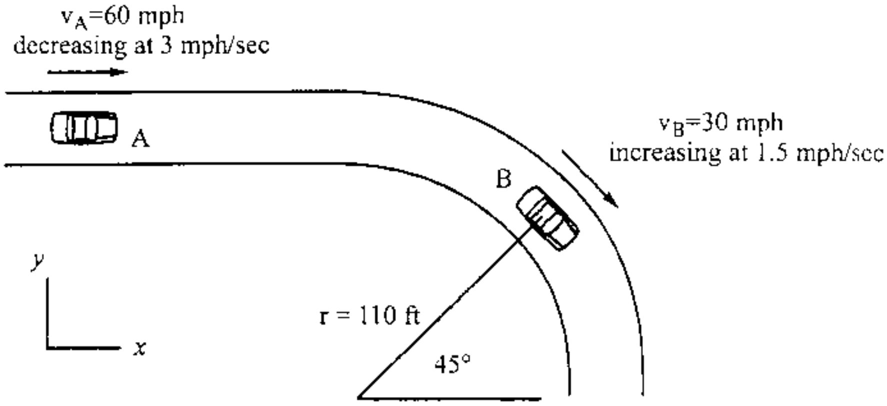
(a) [-6.5 $\boldsymbol{i}$ – 14 $\boldsymbol{j}$] fps^2

(b) [5.7 $\boldsymbol{i}$ + 11.1 $\boldsymbol{j}$] fps^2

(c) [-2.5 $\boldsymbol{i}$ – 4 $\boldsymbol{j}$] fps^2

(d) [7.2 $\boldsymbol{i}$ – 12.3 $\boldsymbol{j}$] fps^2

(e) [-3.3 $\boldsymbol{i}$ + 15.8 $\boldsymbol{j}$] fps^2



Applicable Theory and Hints | **Solution**

This is a relative velocity and relative acceleration problem, where the velocity and accelerations are given for each particle (the automobiles). Write these as vectors, and subtract to get the relative term.

Step 1: Establish a coordinate system and write the velocities and accelerations as vectors. An HP 48G or comparable calculator makes it easy to write and add these vectors in polar form.

Unit conversions: (Magnitudes)

v_A = 60 mph·(88 fps/60 mph) = 88 fps

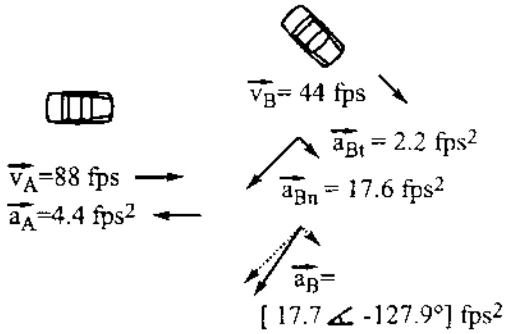
v_B = 30 mph·(88 fps/60 mph) =44 fps

a_A = -3 mph/s·(88 fps/60 mph) = -4.4 fps^2

$a_{Btangential}$ = +1.5 mph/s·(88 fps/60 mph) = 2.2 fps^2

Calculate the normal acceleration of B:

$$a_{Bnormal} = \frac{v^2}{r} = \frac{44^2}{110} = 17.6\,fps^2$$



Write the vectors:

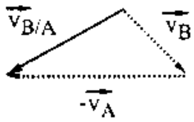
$$\vec{v}_A = \left[88\hat{i}\right] fps = \left[88\,fps\,\angle 0°\right]$$

$$\vec{v}_B = \left[44\,fps\,\angle -45°\right] = \left[31.1\hat{i} - 31.1\hat{j}\right] fps$$

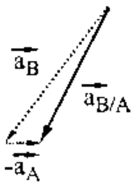
$$\vec{a}_A = \left[-4.4\hat{i}\right] fps^2 = \left[4.4\,fps^2\,\angle 180°\right]$$

$$\vec{a}_B = \left[2.2\,fps^2\,\angle -45°\right] + \left[17.6\,fps^2\,\angle -135°\right]$$
$$= \left[17.74\,fps^2\,\angle -127.9°\right] = \left[-10.9\hat{i} - 14.0\hat{j}\right] fps^2$$

$\overline{v}_{B/A} = \overline{v}_B - \overline{v}_A = \overline{v}_B + (-\overline{v}_A)$



$\overline{a}_{B/A} = \overline{a}_B - \overline{a}_A = \overline{a}_B + (-\overline{a}_A)$



Step 2: Subtract the A terms from the B terms to get the B/A relative terms:

$$\vec{v}_{B/A} = \vec{v}_B - \vec{v}_A = \left[44\,fps\,\angle -45°\right] - \left[88\hat{i}\,fps\right]$$

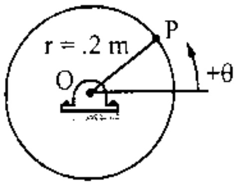
$$\boxed{\vec{v}_{B/A} = \left[-56.9\hat{i} - 31.1\hat{j}\right] fps = \left[64.8\,fps\,\angle -151.3°\right]}$$

$$\vec{a}_{B/A} = \vec{a}_B - \vec{a}_A = \left[17.74\,fps^2\,\angle -127.9°\right] - \left[4.4\,fps^2\,\angle 180°\right]$$

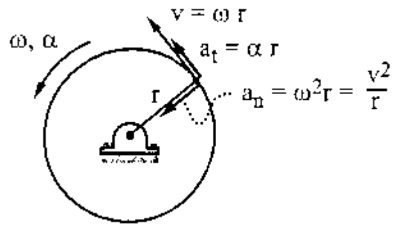
$$\boxed{\vec{a}_{B/A} = \left[15.43\,fps^2\,\angle -114.9°\right] = \left[-6.5\hat{i} - 14.0\hat{j}\right] fps^2}$$

5. The gear shown below starts from rest. The angular position of line OP is given by $\theta = 2t^3 - 7t^2$, where θ is in radians and t in seconds. The magnitude of the total acceleration of point P when t = 2 seconds is: (circle one)

(a) 2.54 m/s^2
(b) 3.18 m/s^2
(c) 3.77 m/s^2
(d) 4.26 m/s^2
(e) 4.39 m/s^2



Basic Theory: Fixed Axis Rotation



Applicable Theory and Hints

General Angular Motion: Definition of Terms

Angular Displacement: θ (radians)

Angular Velocity: $\omega = \dot{\theta} = \frac{d\theta}{dt}$ (radians/sec)

Angular Acceleration: $\alpha = \dot{\omega} = \frac{d\omega}{dt}$ (radians/sec^2)

Eliminate dt: $\alpha \, d\theta = \omega \, d\omega$

For Constant α, Integrated Forms of the Equations:

$$\omega = \omega_0 + \alpha t$$

$$\theta = \theta_0 + \omega_0 t + ½ \alpha t^2$$

$$\omega^2 = \omega_0^2 - 2 \alpha (\theta - \theta_0)$$

Solution at t=2 sec

$\theta = 2t^3 - 7t^2$ $\theta = -12$ rad

$\omega = 6t^2 - 14t$ $\omega = -4$ rad/s

$\alpha = 12t - 14$ $\alpha = +10$ r/s^2

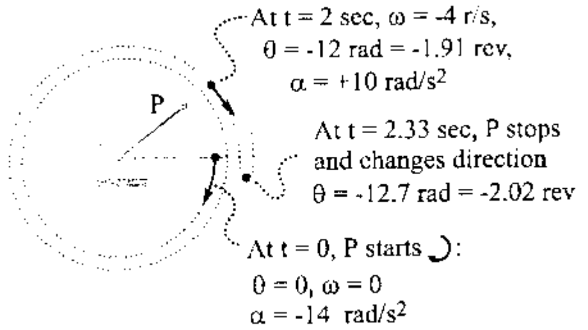
$a_t = \alpha r = .2(10) = 2$ m/s^2

$a_n = \omega^2 r = (-4)^2(.2) = 3.2$ m/s^2

$v = \omega r = (-4)(.2) = .8$ m/s

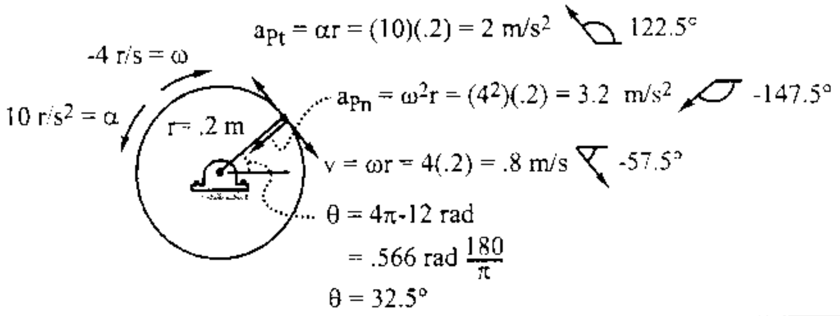
$|a| = \sqrt{a_t^2 + a_n^2} = \sqrt{2^2 + 3.2^2}$

$|a| = 3.77$ m/s^2



At t = 0, the disk begins rotating clockwise (↷) because $\alpha = -14$ rad/s^2. At t = 2 sec, the disk continues rotating clockwise with $\omega = -4$ rad/s, but it is slowing because α is positive. The disk stops momentarily and reverses its direction of rotation at t=2.33 sec. The angular acceleration α remains positive (↶) so the disk will continue its (↶) rotation, gaining angular speed.

Point P: An analysis, at t = 2 sec, of the acceleration components of point P and its velocity is given at right.



6. Below is shown a slider-crank mechanism. If member AB has a constant angular velocity of 3 radians/sec clockwise, the velocity of slider C at the instant shown is: (circle one)

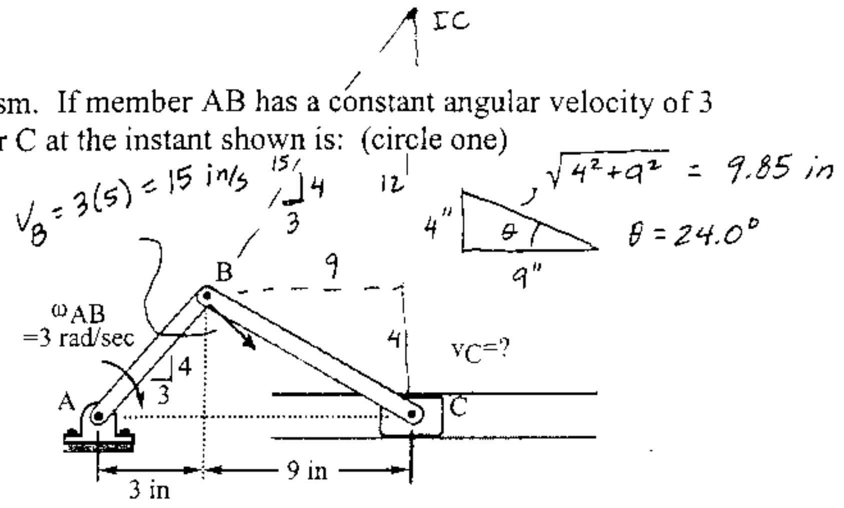
(a) 12 in/sec

(b) 9 in/sec

(c) 16 in/sec

(d) 15 in/sec

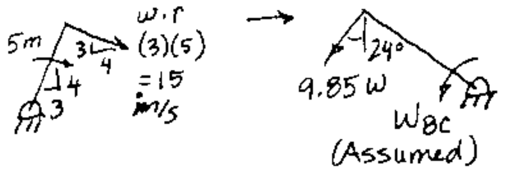
(e) zero (it is momentarily at rest)



$V_B = 3(5) = 15\ in/s$

$\sqrt{4^2+9^2} = 9.85\ in$

$\theta = 24.0°$

Relative Velocity Solution	**Instantaneous Center of Zero Velocity**
$\vec{V}_B = \vec{V}_C + \vec{V}_{B/C}$  $+\downarrow\ \frac{3}{5}15 = 0 + 9.85\omega\cos 24°$ $\boxed{\omega_{BC} = 1.0\ r/s}$ $\xrightarrow{+}\ \frac{4}{5}15 = V_C - 9.85\omega\sin 24$ $V_C = 12 + 4$ $\boxed{V_C = 16\ in/s \rightarrow}$	Any rigid body undergoing general plane motion can be considered, at a given instant, to be rotating about an imaginary "pin"—called the **"instantaneous center of zero velocity (IC)"**—somewhere in the plane. Finding the IC can be helpful for velocity analysis for a rigid body **because it transforms a general plane motion problem—for an instant—into a "fixed" axis rotation problem.** The IC is located by drawing construction lines perpendicular to known velocities on a rigid body. The construction lines intersect at a point of **"zero velocity,"** which acts as a center of rotation. See IC location above for BC. $\omega_{BC} = \frac{V_B}{r_{B/IC}} = \frac{15}{15} = 1\ r/s$ $V_C = r_{C/IC}\cdot\omega_{BC} = (12+4)\cdot 1$ $\boxed{V_C = 16\ in/s \rightarrow}$

7. The wheel shown below rolls without slipping on stationary ground. If the velocity of the center C of the wheel is 2 m/sec to the right, the magnitude of the velocity of point A on the periphery of the wheel is: (circle one)

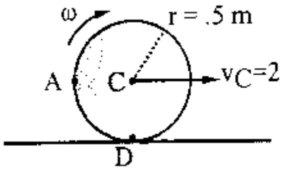
(a) 2 m/s

(b) 2.5 m/s

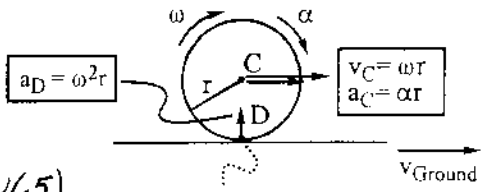
(c) 1.41 m/s

(d) 2.83 m/s

(e) 4 m/s



For a wheel rolling without slipping:



If no slip, $v_D = v_{Ground}$

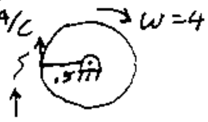
Most often, $\boxed{v_D = 0}$ if $v_G = 0$

Find ω: $V_c = 2 = \omega r = \omega(.5)$

$\omega = 2/.5 = 4\ rad/s$

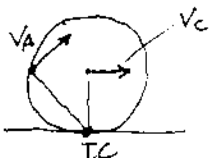
Solution: $\vec{V}_A = \vec{V}_c + \vec{V}_{A/c}$

$\vec{2}$



$\vec{V}_A = [2\hat{i} + 2\hat{j}]\ m/s$

$|V_A| = 2.83\ m/s$



8. Below is shown a mechanism consisting of a rotating disk AB, a link BC, and a slider at C. The wheel AB has a constant angular velocity of 6 radians/sec. At the instant shown, the link BC is translating (its angular velocity is zero). The angular acceleration of link BC is: (circle one)

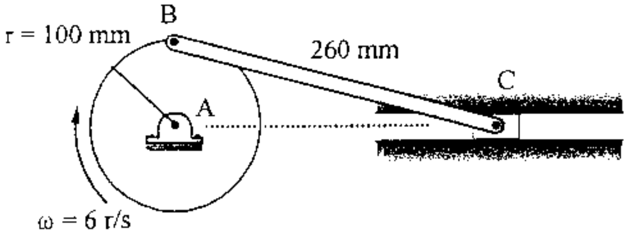
(a) 0

(b) 15 rad/s² CCW

(c) 9 rad/s² CW

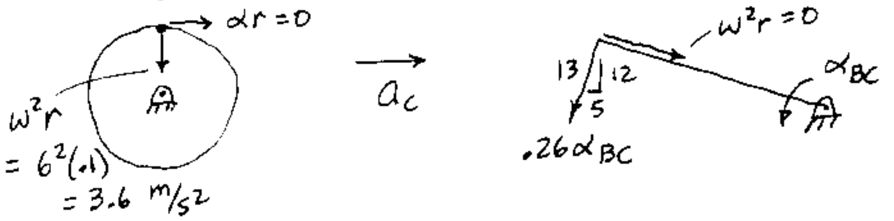
(d) 6 rad/s² CCW

(e) 13 rad/s² CW



Solve using the relative acceleration equation.

$$\vec{a}_B = \vec{a}_c + \left(\vec{a}_{B/C}\right)_{n+t}$$



$+\downarrow \quad 3.6 = 0 + \frac{12}{13}(.26\alpha_{BC})$

$$\boxed{\alpha_{BC} = \frac{3.6}{.24} = 15 \text{ r/s}^2 \circlearrowleft}$$

$\xrightarrow{+} \quad 0 = a_c - \frac{5}{13}(.26\alpha)$

$$\boxed{a_c = .10(15) = 1.5 \text{ m/s}^2 \rightarrow}$$

9. The 10 kg ball is supported by a cord and swings in the vertical plane. At the instant shown, the velocity of the ball is 3 m/s and the tension in the cord is: (circle one)

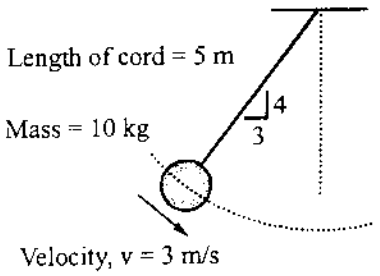
(a) 49 N

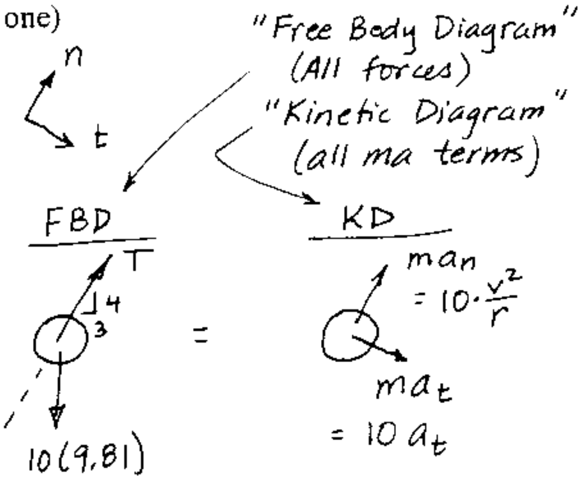
(b) 58.9 N

(c) 67.5 N

(d) 78.5 N

(e) 96.5 N





This is a particle F = ma problem in n-t coordinates.

Equations: $\Sigma F_n = ma_n$; $\Sigma F_t = ma_t$; $a_n = v^2/\rho$

$+\nearrow \quad \Sigma F_n = ma_n; \quad T - \frac{4}{5}(10)(9.81) = 10\frac{(3)^2}{5}$

$$\boxed{T = 18 + 78.5 = 96.5 \text{ N}}$$

$\overset{+}{\searrow} \quad \Sigma F_t = ma_t; \quad \cancel{10}(9.81)\frac{3}{5} = \cancel{10}a_t$

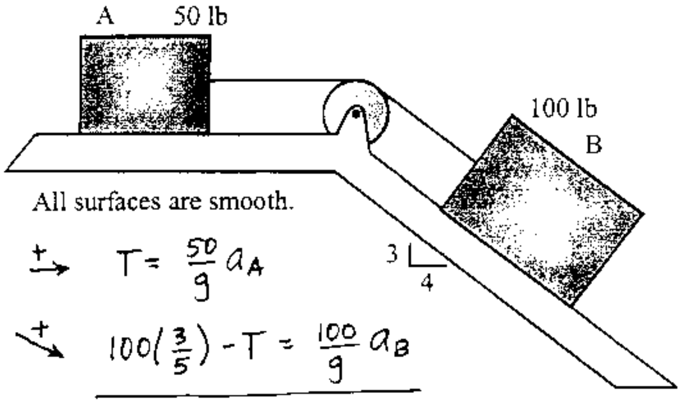
$$\boxed{a_t = 5.89 \text{ m/s}^2 \ {}_{3}\!\!\lrcorner_{4} \searrow}$$

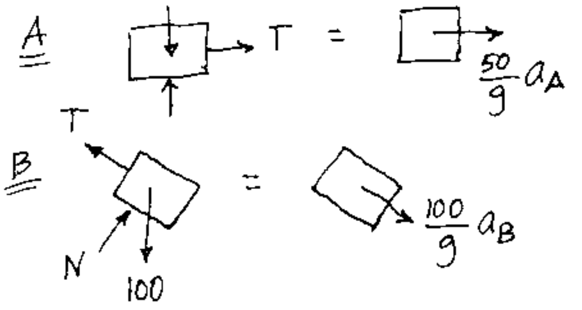
10. The two blocks shown below are connected by an inextensible (cannot stretch) cord and are free to move on frictionless surfaces. The pulley is frictionless and massless. When the system is released, the tension in the cord is: (circle one)

(a) 20 lb (d) 80 lb

(b) 50 lb (e) 110 lb

(c) 60 lb





A $\xrightarrow{+}$ $T = \frac{50}{g} a_A$

B ↘+ $100(\frac{3}{5}) - T = \frac{100}{g} a_B$

$60 = \frac{150}{g} a$

(Kinematics: $a_A = a_B = a$)

$\boxed{a = \frac{2}{5} g = 12.88 \text{ fps}^2}$

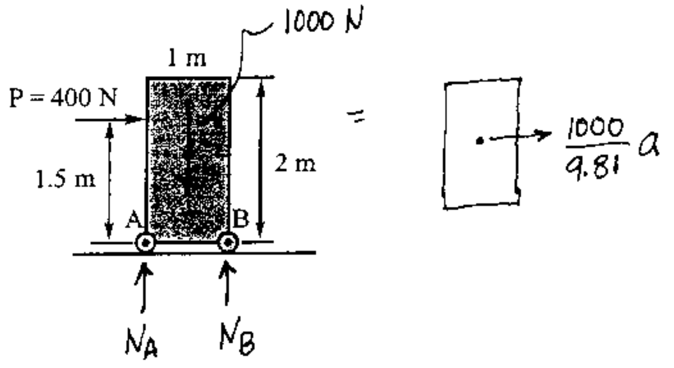
$T = \frac{50}{\cancel{g}} \cdot \frac{2}{5} \cancel{g} = \boxed{20 \text{ lb} = T}$

11. The homogeneous 1000 Newton crate rests on small frictionless rollers of negligible mass. When a 400 N force is applied to the crate as shown, the combined normal reaction force on the front rollers at B is: (circle one)

(a) 400 N (d) 1100 N

(b) 500 N (e) 0 N (i.e. it is tipping)

(c) 700 N



Rigid Body F=ma

Eqns: $\Sigma F_x = m a_x$

$\Sigma F_y = m a_y$

$\left\{ \begin{array}{l} \Sigma M_G = I_G \alpha \\ \text{or} \\ \Sigma M_P = \Sigma (M_K)_P \end{array} \right.$

This is a translation problem, i.e. where $\alpha = 0$. Assume this, then inspect N_A + N_B results to see if they are consistent with $\alpha = 0$.

$\xrightarrow{+}$ $400 = \frac{1000}{9.81} a$; $a = .4g$
$= 3.92 \text{ m/s}^2$

$+\uparrow \Sigma F_y = m a_y$; $N_A + N_B - 1000 = 0$
$N_A + N_B = 1000$

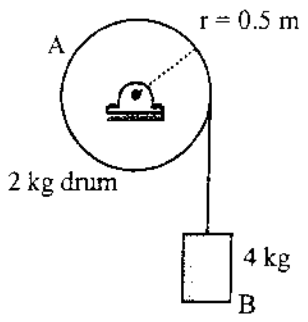
$+\circlearrowleft \Sigma M_G = I_G \alpha = 0$; $400(.5) + N_A(.5) - N_B(.5) = 0$
$N_A - N_B = -400$

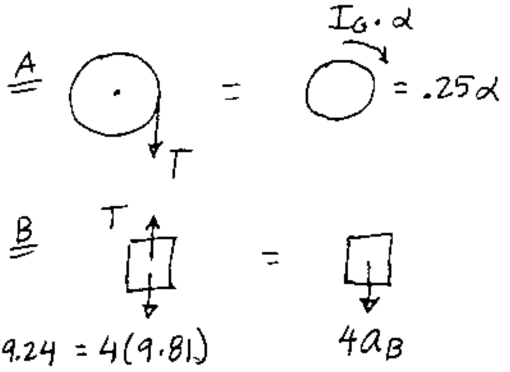
$2N_A = 600 \text{ N}$

$\boxed{\begin{array}{l} N_A = 300 \text{ N} \\ N_B = 700 \text{ N} \end{array}}$

12. A 4 kg mass B is suspended by a slender cable which wraps around a 2 kg drum A. When the system is released to move, the tension in the cable is: (circle one)

(a) 7.85 N
(d) 39.24 N
(b) 23.5 N
(e) 47.09 N
(c) 31.39 N





Disk: $I_G = \frac{1}{2}mr^2$

(Mass Moment of Inertia)

$I_G = \frac{1}{2}(2)(.5)^2 = .25 \text{ kg}\cdot\text{m}^2$

Kinematics: $a_B = r\alpha = .5\alpha$

$39.24 = 2.5\alpha$

$\boxed{\alpha = 15.7 \text{ r/s}^2 \curvearrowright}$

$\boxed{T = .5\alpha = 7.85 \text{ N}}$

A $\overset{+}{\curvearrowleft}$ $\Sigma M_G = I_G\alpha$; $T(.5) = .25\alpha$

$T = .5\alpha$

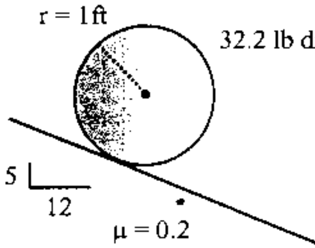
B $+\downarrow$ $39.24 - T = 4a_B$

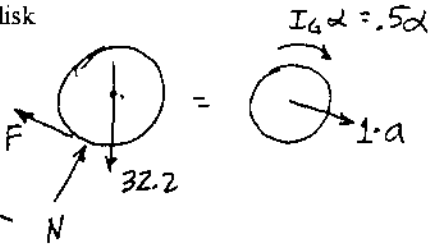
$39.24 = .5\alpha + 4a_B$

13. The 32.2 lb homogenous cylinder is released from rest on the inclined plane shown below. Its angular acceleration will be: (circle one)

$I_G = \frac{1}{2}\left(\frac{32.2}{32.2}\right)1^2 = .5 \text{ slug}\cdot\text{ft}^2$

(a) 13.4 rad/s^2
(d) 5.9 rad/s^2
(b) 12.4 rad/s^2
(e) 8.3 rad/s^2
(c) 3.2 rad/s^2





This is a general plane motion problem ($F = ma$).

Important! You cannot automatically set $F = \mu N$, unless the problem specifies slipping.

$F \leq \mu N$ is an inequality.

Need kinematics or another eqn.

① Assume No slip: ∴ $a = r\alpha$

$a = 1 \cdot \alpha$

or ② Assume slip: ∴ $F = \mu N$

Whichever you assume, you must check the assumption. The check for ① is easier.

check: $\left(\frac{F}{N}\right)_{CALC} \overset{?}{\leq} \mu$ $\quad \frac{4.13}{29.7} = .14 < .2$!

$\boxed{F = .5\alpha = 4.13 \text{ lb}}$

$+\nearrow$ $N - \frac{12}{13}32.2 = 0$

$N = 29.72$ lb

$\overset{+}{\searrow}$ $\frac{5}{13}32.2 - F = a$

$12.39 - F = a$

$+\curvearrowleft \Sigma M_G = I_G\alpha$; $F(1) = .5\alpha$

$F = .5\alpha$

$12.39 = a + .5\alpha$

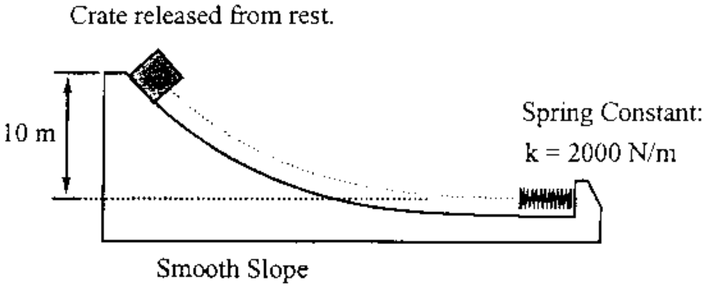
Assume No Slip: $a = r\alpha = 1 \cdot \alpha$

$12.39 = 1.5\alpha$

$\boxed{\alpha = 8.26 \text{ r/s}^2 \curvearrowright}$

14. A 5 kg crate is released from rest on the smooth slope shown below. The crate slides down the slope, gaining speed, until it strikes the originally unstretched spring (k = 2000 N/m). When the crate comes to a stop, the spring will have compressed: (circle one)

(a) .3 m
(b) .4 m
(c) .5 m
(d) .6 m
(e) .7 m



Particle Work-Energy Eqn:

$$\underbrace{T_1}_{\text{Initial KE} = \frac{1}{2}mv_1^2} + \underbrace{\sum U_{1-2}}_{\text{Work}} = \underbrace{T_2}_{\text{Final KE}}$$

Work: Force · Distance

Spring: $-\frac{1}{2}k(s_2^2 - s_1^2)$

$$\cancel{T_1}^{0} + \sum U_{1-2} = \cancel{T_2}^{0}$$

$$+(10m)(5kg)(9.81) - \frac{1}{2}(2000)x^2 = 0$$

$$\boxed{x = 0.70\ m}$$

15. A 2 m long, 10 kg slender rod AB is released from rest in the horizontal position. It swings counter-clockwise in the vertical plane, pivoting about a smooth pin at A. When it reaches the vertical position, its angular velocity ω is: (circle one)

(a) 1.2 rad/s
(b) 7.7 rad/s
(c) 19.6 rad/s
(d) 9.8 rad/s
(e) 3.8 rad/s

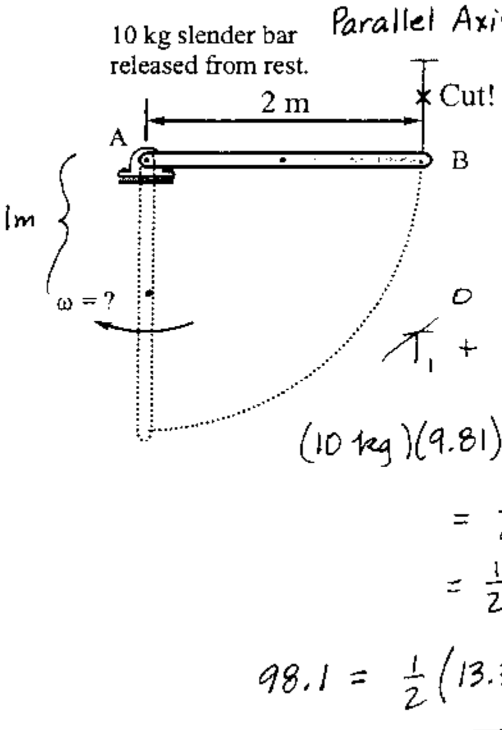
Slender Bar: $\boxed{I_G = \frac{1}{12}mL^2}$

Parallel Axis Thm: $I_A = I_G + md$

$d = L/2$

$= \frac{1}{12}mL^2 + m(\frac{L}{2}$

$\boxed{I_A = \frac{1}{3}mL^2}$



R Body W-E Eqn

$$\sum U_{1-2} = T_2$$

Wor : • Force · Distance

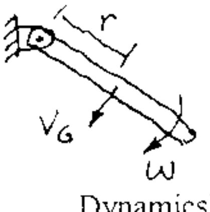
- Spring: $-\frac{1}{2}k(s_2^2 - s_1^2)$
- Couple: $M \cdot \theta$ (θ in radians)

KE Storage in a Rigid Body

Generally, $\boxed{T = \frac{1}{2}mv_G^2 + \frac{1}{2}I_G\omega^2}$

Translation Case: $T = \frac{1}{2}mv_G^2$

Rotation (about pin at G): $T = \frac{1}{2}I_G\omega^2$



$$T = \frac{1}{2}mv_G^2 + \frac{1}{2}I_G\omega^2$$

but $v_G = \omega \cdot r$

$$T = \frac{1}{2}(I_G + mr^2)\omega^2 \Rightarrow \boxed{T = \frac{1}{2}I_{pin}\omega^2}$$

$$\cancel{T_1}^{0} + U_{1-2} = T_2$$

$$(10\ kg)(9.81)(1m) = \frac{1}{2}I_{pin}\,\omega^2 = \frac{1}{2}\left[\frac{1}{3}(10)(2)^2\right]\omega^2$$

$$98.1 = \frac{1}{2}(13.33)\,\omega^2$$

$$\omega = \sqrt{14.71}$$

$$\boxed{\omega = 3.84\ r/s}$$

16. Two identical cylinders R and S are released simultaneously from rest on the top of two identical inclined planes having the same length and slope. Cylinder R (for "rough") rolls without slipping on a slope with some friction, while cylinder S (for "smooth") rolls down a perfectly smooth plane. The two cylinders reach the bottom of their respective planes (or "hills"): (circle one)

(a) at the same instant

(b) with the same angular velocity

(c) with the same linear velocity of the mass centers

(d) with the same kinetic energy

(e) with none of the above

17. Immediately after being driven off a tee by a golf club, a 45 gram golf ball leaves the club face with a velocity of 80 m/s. If the time of impact between the club and the ball is 10 milliseconds, determine the average force developed between the club and the ball during impact. (circle one)

(a) 45 N	(d) 100 N
(b) 200 N	(e) 360 N
(c) 440 N	

Impulse - Momentum Eqn

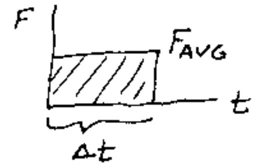
$$mV_1 + \int F\,dt = mV_2$$

$$\overset{0}{\cancel{mV_1}} + F_{AVG} \cdot \Delta t = mV_2$$

$$F_{AVG}\,(.010\text{ sec}) = (.045)(80\text{ m/s})$$

$$\boxed{F_{AVG} = 360\text{ N}}$$

Assumes



18. Blocks A and B have weights and initial velocities as shown and move on a smooth surface. The velocity of block B immediately after impact is observed to be 4 fps to the right. The velocity of cart A immediately after impact is: (circle one)

(a) 1 fps	(d) 5.6 fps
(b) 2 fps	(e) 10 fps
(c) 4 fps	

Blocks A and B weigh 20 lb and 30 lb, respectively. The surface is smooth.

Cons. of Linear Momentum

$$\Sigma(m\vec{v})_1 = \Sigma(m\vec{v})_2$$

$$\xrightarrow{+} \quad \frac{20}{g}(+7) + \frac{30}{g}(-2) = \frac{20}{g}V_{A2} + \frac{30}{g}\overset{+4}{V_{B2}}$$

$$140 - 60 = 80 = 20V_{A2} + 120$$

$$V_A = \frac{-40}{20} = -2\text{ fps}$$

$$\boxed{V_A = 2\text{ fps} \leftarrow}$$

19. The carts in problem 16 above rebound from the impact with a coefficient of restitution, e, of: (circle one)

(a) 0.67	(d) 1.2
(b) 0.22	(e) 2.0
(c) 0.40	

$$\xrightarrow{+} \quad e = \frac{V_{B2} - V_{A2}}{V_{A1} - V_{B1}}$$

$$e = \frac{(4) - (-2)}{7 - (-2)} = \frac{6}{9} = 0.667$$